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Tax Compliance and Rank-Dependent Expected Utility

Abstract

In this paper, we show that considering the classic Allingham and Sandmo (1972) tax compliance problem under Rank-Dependent Expected Utility (RDEU) axiomatics provides a simple explanation for the "excess" level of full compliance observed in empirical studies, and which standard Expected Utility (EU) axiomatics are unable to explain. RDEU axiomatics provide a compelling answer to this puzzle, without the need for the moral sentiments or stigma arguments that have recently been advanced in the literature. Formally, we show that the threshold audit probability at which full compliance becomes optimal for the consumer is significantly lower under RDEU axiomatics than in the EU case. We also show that the comparative statics of tax-evasion with respect to changes in the tax rate or in income are "weaker" than under EU axiomatics. We conclude by presenting numerical simulations using several different parameterizations of the probability weighting function that have been proposed in the literature.

Keywords: Rank-dependent expected utility, tax-compliance

JEL: D81, H26.

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1 Introduction

Since the seminal article by Allingham and Sandmo (1972), most authors dealing with the issue of tax compliance have had great difficulty in making their theoretical models square with empirical or experimental results. The most glaring example is the tendency of many individuals to engage in *no* tax evasion at all.

A first common reaction has been to "improve" upon the initial model. Yitzhaki's (1974) contribution was to assume that the penalty for non-compliance is proportional to the amount of taxes evaded, while Pencavel (1979) endogenized income by jointly considering labor supply along with tax compliance. Koskela (1983) considers the nature of penalty schemes (charged on undeclared income or on the amount of tax evaded). Pestieau and Possen (1991), for their part, included the choice of activity by the consumer, where each sector varies in the opportunities available for evasion. Engel and Hines (1994) focused upon the repeated nature of the problem and explored its dynamics. Finally Graetz, Reinganum and Wilde (1986), and Beck and Jung (1989) introduced strategic concerns: the former endogenized the audit probability within a principal-agent framework, in which the audit probability becomes a function of the amount of income declared; the latter used game-theoretic tools to model the interaction between the taxpayer and the tax authorities.

While these developments have been both interesting and important, most observers of the literature agree that they still do not allow one to reconcile theory with observed empirics (see e.g. the survey by Andreoni, Erard and Feinstein, 1998). As a result, a number of authors have introduced new constraints, derived from psychological arguments, in an effort to explain the "excessive" level of observed compliance. Spicer and Lundstedt (1976), for example, considered the degree of satisfaction felt by the taxpayer with respect to his government. Erard and Feinstein (1994) included notions of guilt and shame in the taxpayer's objective function. While the heuristic appeal of these arguments is undeniable, they remain, however, difficult to justify on purely economic grounds.

A second approach has been to raise doubts concerning the expected utility framework initially adopted by Allingham and Sandmo (1972). Slemrod (1992), for example, summarizes a large corpus of empirical and experimental literature that finds a subjective probability of audit that is significantly different from (and larger than) the observed objective probabilities. Our paper picks up on this idea as its point of departure, and takes aim at the fundamental building-block of the Allingham and Sandmo (1972) approach: the expected utility (EU) model of von Neumann and Morgenstern. As such, we formalize the Allingham and Sandmo (1972) problem under alternative axiomatics, using the Rank-Dependent Expected Utility (RDEU) model.[1]

The RDEU approach was initially developed by Quiggin (1982) in order to address a number of important weaknesses that had become apparent in the EU approach.[2] Under RDEU axiomatics, the linearity in probabilities of the EU model is replaced by a weighting function which assigns weights to the probabilities of the different states of nature, where the weights are themselves functions of the rank of the given state of nature, in terms of the level of satisfaction that the individual derives.

As applied to tax compliance, we prefer the RDEU approach to its main competitor, Cumulative Prospect Theory (CPT), developed in a series of papers beginning with Kahneman and Tversky (1979). CPT does, as noted by Cowell (2003), present a number of advantages. First, individuals "edit" the information associated with the underlying lotteries. Second, and contrary to the RDEU approach, CPT allows one to distinguish the value function (as opposed to the utility function) from the weighting function associated with the probabilities

[1] Our paper therefore constitutes a response to the question posed by Cowell (2003, p. 9): "Would relaxing this assumption to encompass non-EU models -such as rank-dependent utility or prospect theory- result in a more promising underlying story?"

[2] While the Allais (1953) paradox had already revealed significant deficiencies in terms of the explanatory power of the EU approach in risky situations and, in particular, calls the Independence Axiom into question (as noted by Kahneman and Tversky, 1979), Savage's construct was unable to account for the Ellsberg paradox (1961). In addition, the EU approach results in the utility function playing two roles: on the one hand it describes the decision-maker's attitudes toward risk (risk aversion flows from the concavity of the utility function); on the other, it describes an individual's preferences in situations in which risk is absent (concavity simply denotes decreasing marginal utility). The EU approach is therefore incompatible with a situation in which, for example, an individual would have decreasing marginal utility but a certain taste for risk-taking. Finally, the EU model does not allow one to distinguish between optimistic and pessimistic decisionmakers. All of these limitations are addressed by the RDEU approach (see e.g. Starmer, 2000).

of given gains or losses. On the other hand, and in the context of tax compliance we believe this argument to be the clincher, CPT imposes a reference point.[3] This last hypothesis implies that consumers should be indifferent to wealth effects, which runs counter to all empirical evidence that indicates that the degree of fraud is positively correlated with the consumer's level of income.

Finally, note that Bernasconi (1998) analyzes tax compliance using the notion of first-order risk-aversion, introduced into the literature by Segal and Spivak (1990). In a two states of nature example (which corresponds exactly to the Allingham and Sandmo tax compliance problem) the particularity of Segal and Spivak's approach is that an individual's indifference curves possess a kink along the 45 degree line (which corresponds to perfect insurance). Formally, individual preferences admit points of non-differentiability, where risk-aversion is of order one. This property arises naturally under RDEU axiomatics. Indeed, Bernasconi (1998) illustrates his results using a numerical simulation based on a RDEU model in which the parameterization is borrowed from the empirical work of Camerer and Ho (1994). Our paper can thus be seen as a natural extension to Bernasconi's work, in which RDEU axiomatics are posed both explicitly and right from the start.

The paper is organized as follows. In part 2 we introduce the basic notation that we use and express the Allingham and Sandmo problem in terms of RDEU axiomatics. We then present our main theoretical result which shows that full compliance is "easier" to obtain under RDEU axiomatics than under EU axiomatics. Formally speaking, this is expressed by the fact that the threshold audit probability at which full compliance becomes optimal for the consumer is lower under RDEU axiomatics than in the EU case. A Corollary establishes that the comparative statics of cheating with respect to changes in the tax rate or in income are "weaker" than under EU axiomatics. We then perform numerical simulations using the parameterizations of the probability weighting function proposed by Camerer and Ho (1994), Tversky and Fox (1995) and Prelec (1998), and examine: (*i*) the minimal penalty

[3] This hypothesis translates the existence of a status quo, and corresponds to the normalization that $u(0) = 0$.

rate needed to ensure full compliance, (ii) the proportion of income that is declared. These simulations show that RDEU axiomatics combined with the Prelec (1998) parameterization of the probability weighting function provide a coherent explanation of the tax-compliance puzzle.

2 Allingham-Sandmo (1972) under RDEU Axiomatics

2.1 Basic notation

In the RDEU approach, the independence axiom of expected utility theory is replaced by the axiom of the comonotone sure thing (Chateauneuf , 1999). The set X of riskless alternatives is a nonempty compact topological space. A lottery is a probability measure with finite support on X, typically denoted by $P = (p_1, x_1; ...; p_i, x_i; ...; p_n, x_n)$ where $x_1, ..., x_n \in X$, $p_i \geqslant 0$ for all i, and $\sum_{i=1}^{n} p_i = 1$. The set of all lotteries is $L(X)$. A riskless alternative $x \in X$ is identified with the lottery $(1, x) \in L(X)$.

The preferences of an RDEU decisionmaker are characterized by two functions, u and φ, that are continuous and increasing. The function $u : \mathbf{R} \longrightarrow \mathbf{R}$ defined up to a monotone increasing transformation, plays the role of a utility function under certainty; the function $\varphi : [0,1] \longrightarrow [0,1]$, which satisfies the restrictions $\varphi(0) = 0$ and $\varphi(1) = 1$, is unique and plays the role of a probability transformation function. For a decision-maker with utility function $u(.)$, preferences over $L(X)$ can be modelled by RDEU if there exists a weighting function $\varphi(.)$, such that preferences are represented by the functional $E_{RDEU}[.] : L(X) \longrightarrow \mathbf{R}$ defined by:

$$E_{RDEU}[P] = \sum_{j=1}^{n-1} \left(\varphi\left(\sum_{i=j}^{n} p_{\rho(i)} \right) - \varphi\left(\sum_{i=j+1}^{n} p_{\rho(i)} \right) \right) u\left(x_{\rho(j)}\right) + \varphi\left(p_{\rho(n)}\right) u\left(x_{\rho(n)}\right),$$

where $\rho(.)$ is a permutation that orders the riskless alternatives in the lottery from worse to best, i.e. $u(x_1) \leqslant ... \leqslant u(x_n)$ (Quiggin, 1982).[4]

2.2 The model

In the Yitzhaki (1974) version of the standard Allingham and Sandmo (1972) problem, the penalty faced by the taxpayer in the case of an audit is proportional to the amount of tax avoided, when the taxpayer engages in a positive amount of avoidance. Consider the lottery $P(z) = (p, \widetilde{y} - \theta tz; 1-p, \widetilde{y} + tz)$, where p is the probability of being audited, $\widetilde{y}$ is the taxpayer's after tax income, t is the tax rate, θ is the penalty rate if fraud is detected, and z is the amount of underreporting by the taxpayer[5]. The taxpayer's objective function is given by

$$RDEU[P(z)] = \varphi(1-p)\, u(\widetilde{y} + tz) + (1 - \varphi(1-p))\, u(\widetilde{y} - \theta tz). \tag{1}$$

The solution to the taxpayer's optimization problem when her preferences are described by RDEU axiomatics is given by:

$$z^*_{RDEU} \equiv \arg\max_{z \geqslant 0} RDEU[P(z)]. \tag{2}$$

The necessary First Order Condition (FOC) which characterizes the solution defined in (2) is given by:

$$t\left[\varphi(1-p)\, u'\left(\widetilde{y} + tz^*_{RDEU}\right) - \theta\left[1 - \varphi(1-p)\right] u'\left(\widetilde{y} - \theta tz^*_{RDEU}\right)\right] + \lambda = 0, \tag{3}$$

[4] We present the definition of $E_{RDEU}[P(z)]$ in terms of a decumulative density function (i.e., 1 minus the cumulative density), whereas, in his illustration of the kink, Bernasconi (1998) uses a cumulative density function formulation.

[5] Note that we assume $z \geqslant 0$. If one had $z < 0$, the structure of the optimization program implies that the taxpayer would receive a reward for over-declaration (this follows because of the formulation in terms of a penalty: $-\theta tz^* > 0$ if $z^* < 0$). As such, we prefer to assume, as in Andreoni *et al* (1998) that overdeclaration is irrational.

where λ is the Lagrange multiplier associated with the constraint $z \geqslant 0$. Since the Second Order Condition (SOC) is given by:

$$t^2 \left[\varphi(1-p)\, u''\left(\widetilde{y} + t z^*_{{}_{RDEU}}\right) + \theta^2 \left[1 - \varphi(1-p)\right] p u''\left(\widetilde{y} - \theta t z^*_{{}_{RDEU}}\right)\right] < 0,$$

which always holds because of the strict concavity of the utility function $u(.)$, (3) is sufficient as well as being necessary. From Kuhn-Tucker, complementary slackness implies $\lambda z^*_{{}_{RDEU}} = 0$. Now rewrite (3) as:

$$-\lambda = t\theta\left[1 - \varphi(1-p)\right] u'\left(\widetilde{y} + t z^*_{{}_{RDEU}}\right)\left[\frac{\varphi(1-p)}{\theta\left[1 - \varphi(1-p)\right]} - \frac{u'\left(\widetilde{y} - \theta t z^*_{{}_{RDEU}}\right)}{u'\left(\widetilde{y} + t z^*_{{}_{RDEU}}\right)}\right]. \tag{4}$$

Two cases will arise because of complementary slackness. First, we may have $z^*_{{}_{RDEU}} = 0$, in which case $\lambda > 0$. Rearranging (4) implies that:

$$-\lambda = t\theta\left[1 - \varphi(1-p)\right] u'(\widetilde{y})\left[\frac{\varphi(1-p)}{\theta\left[1 - \varphi(1-p)\right]} - 1\right] < 0,$$

which can only be true for $(1+\theta)\varphi(1-p) - \theta < 0$, but is impossible because $(1+\theta)\varphi(1-p) - \theta > 0$. Second, we may have $z^*_{{}_{RDEU}} > 0$, in which case $\lambda = 0$ and (4) can be rewritten as:

$$\frac{\varphi(1-p)}{\theta\left[1 - \varphi(1-p)\right]} = \frac{u'\left(\widetilde{y} - \theta t z^*_{{}_{RDEU}}\right)}{u'\left(\widetilde{y} + t z^*_{{}_{RDEU}}\right)} \equiv f\left(z^*_{{}_{RDEU}}; \widetilde{y}, t, \theta\right). \tag{5}$$

By inspection of (5) it is immediate that $f_{z^*}\left(z^*_{{}_{RDEU}}; .\right) > 0$. Moreover, it is equally clear that $f(0; .) = 1$. Therefore (5) cannot hold when $(1+\theta)\varphi(1-p) - \theta < 0$ and can only obtain when $(1+\theta)\varphi(1-p) - \theta \geqslant 0$. By the Theorem of the Maximum, $z^*_{{}_{RDEU}}$ as implicitly defined by (5) is continuous in p. More formally, consider the partial inverse of f with respect to $z^*_{{}_{RDEU}}$, which we shall denote by ψ, where $\psi\left(f\left(z^*_{{}_{RDEU}}; \widetilde{y}, t, \theta\right); \widetilde{y}, t, \theta\right) = z^*_{{}_{RDEU}}$. It follows that $\psi(f(0; .); .) = \psi(1; .) = 0$. Since f is increasing, so is its inverse: $\psi_f(f; .) > 0$. From the preceding discussion:

$$z^*_{RDEU}\left(p;\widetilde{y},t,\theta\right)=\begin{cases}\psi\left(\frac{\varphi(1-p)}{\theta[1-\varphi(1-p)]};\widetilde{y},t,\theta\right), \varphi\left(1-p\right)-\theta\left[1-\varphi\left(1-p\right)\right]\geqslant 0\\ 0,\quad otherwise\end{cases}\tag{6}$$

If we pose $\varphi\left(p\right)=p$ for every $p\in[0,1]$, then we are back to Expected Utility axiomatics, and one obtains the following result:

$$z^*_{EU}\left(p;\widetilde{y},t,\theta\right)=\begin{cases}\psi\left(\frac{1-p}{\theta p};\widetilde{y},t,\theta\right), 1-p-\theta p\geqslant 0\\ 0,\quad otherwise\end{cases}\tag{7}$$

This result is a standard one in the tax-compliance literature (see e.g. Andreoni *et al* (1998)).

Since $\varphi\left(.\right)$ is a strictly increasing function from $[0,1]$ to $[0,1]$, its inverse $\varphi^{-1}\left(.\right)$ is so as well and the condition $\varphi\left(1-p\right)-\theta\left[1-\varphi\left(1-p\right)\right]\geqslant 0$ can be rewritten as:

$$p\leqslant 1-\varphi^{-1}\left(\frac{\theta}{1+\theta}\right)=\underline{p}^*_{RDEU}\left(\theta\right).$$

We then have the following Proposition:

Proposition 1 *The taxpayer's optimal compliance behavior is given by:*

$$z^*_{RDEU}\left(p;\widetilde{y},t,\theta\right)=\begin{cases}\psi\left(\frac{\varphi(1-p)}{\theta[1-\varphi(1-p)]};\widetilde{y},t,\theta\right),\quad p<\underline{p}^*_{RDEU}\left(\theta\right)\\ 0,\quad otherwise\end{cases}\tag{8}$$

Experimental studies show that the probability weighting function $(\varphi\left(.\right))$ is inverse S-shaped (first concave, then convex), overweighting low probabilities and underweighting high probabilities (Tversky and Kahneman,1991, Abdellaoui, 2000). The probability weighting function therefore satisfies the condition that:

$$\exists\ \widehat{p}\in\left[0,1\right],\quad\text{such that}\quad\varphi\left(\widehat{p}\right)=\widehat{p}.$$

Moreover, we have:

$$\begin{cases} \forall p < \widehat{p} & \varphi(p) > p \\ \forall p > \widehat{p} & \varphi(p) < p \end{cases}$$

For Prelec (1998), "the overweighting of small probabilities, below the fixed point ($\widehat{p}$), enhances the attraction of small-p gains (lottery tickets) and the aversion to small-p losses (audit), while the underweighting of larger probabilities above the fixed point, diminishes the attraction of larger-p gains (underdeclaration without auditing) and the aversion to larger-p losses." Prelec (1998) also establishes that $\widehat{p}$, the fixed point of $\varphi(.)$, lies between 0.2 and 0.4. Decidue and Wakker (2001) specify that: "Descriptively, a pessimistic attitude can result from irrational belief that unfavorable events tend to happen more often, leading to an unrealistic overweighting of unfavorable likehoods (Murphy's law)." In what follows, we assume:

Condition 1

$$\varphi(1-p) < 1-p$$

Condition 1 boils down to assuming one of three things. First, $\varphi(.)$ may be S-shaped and the audit probability p is strictly smaller than the fixed point $\widehat{p}$, an hypothesis that would appear reasonable in light of the available estimates of the audit probability ($p \in [0.01, 0.05]$) (Andreoni *et al*, 1998) and of the fixed point ($\widehat{p} \in [0.2, 0.4]$). Second, when Condition 1 is given by $\varphi(1-p) \leqslant 1-p$, the individual may be weakly risk-averse in the sense of Quiggin (1982) and Yaari (1987). According to these authors, a RDEU decisionmaker is said to display weak risk-aversion if and only if $u(.)$ is concave and for all $\pi \in [0,1]$, $\varphi(\pi) \leqslant \pi$, where π is the probability of a favorable event. When the taxpayer cheats, an audit is an unfavorable event, since it results in the taxpayer paying a penalty. The favorable event therefore obtains with probability $1-p$. Third, and again for $\varphi(1-p) \leqslant 1-p$, the individual may be strongly risk-averse (which corresponds to an aversion to mean-preserving spreads) in the sense of Chew, Karni and Safra (1987), which obtains when $u(.)$ is concave and $\varphi(.)$

convex. Note that the third assumption is stronger than the second, which in turn is stronger than the first.

A second PROPOSITION obtains when one compares $z^*_{EU}(p;\widetilde{y},t,\theta)$ and $z^*_{RDEU}(p;\widetilde{y},t,\theta)$:

Proposition 2 *Consider* $z^*_{EU}(p;\widetilde{y},t,\theta)$ *and* $z^*_{RDEU}(p;\widetilde{y},t,\theta)$ *as defined in equations (7) and (8), and a probability weighting function which respects* CONDITION 1*. Then:*

$$\underline{p}^*_{RDEU}(\theta) < \underline{p}^*_{EU}(\theta),$$

and

(*i*) *for* $p < \underline{p}^*_{RDEU}(\theta), \quad z^*_{EU}(p;.) > z^*_{RDEU}(p;.) > 0;$

(*ii*) *for* $\underline{p}^*_{RDEU}(\theta) \leqslant p < \underline{p}^*_{EU}(\theta), \quad z^*_{EU}(p;.) > z^*_{RDEU}(p;.) = 0;$

(*iii*) *for* $\underline{p}^*_{EU}(\theta) \leqslant p, \quad z^*_{EU}(p;.) = z^*_{RDEU}(p;.) = 0.$

Proof. Assume CONDITION 1, $\varphi(1-p) < 1-p$. Rewrite this as $1-p-\varphi(1-p) > 0$. Adding $p\varphi(1-p)$ to both sides yields $1-p-\varphi(1-p)+p\varphi(1-p) > p\varphi(1-p)$, which can be factorized as $(1-p)[1-\varphi(1-p)] > p\varphi(1-p)$. Rearranging this inequality and dividing both sides by θ yields:

$$\underline{p}^*_{EU}(\theta) = \frac{1-p}{\theta p} > \frac{\varphi(1-p)}{\theta[1-\varphi(1-p)]} = \underline{p}^*_{RDEU}(\theta).$$

Since $\psi_f(f;.) > 0$, it follows that:

$$\psi\left(\frac{1-p}{\theta p};.\right) > \psi\left(\frac{\varphi(1-p)}{\theta[1-\varphi(1-p)]};.\right),$$

which implies that $z^*_{EU}(p;.) > z^*_{RDEU}(p;.)$. The rest of PROPOSITION 2 is immediate. ■

Pessimism is characterized by a convex weighting function. Similarly, optimism corresponds to a concave weighting function (Quiggin, 1982). In the original contribution by Allingham and Sandmo (1972), the main factor limiting tax avoidance is the consumer's risk-aversion; RDEU axiomatics allow one to add pessimism to the picture, in the sense of the consumer's overweighting of lower-ranked outcomes (in this case, being audited). The pessimism of individuals leads them to a greater degree of compliance than in the EU case.

We now assume the following:

Condition 2

$$A(.) = -\frac{u''(.)}{u'(.)} \text{ is decreasing.}$$

Under EU axiomatics, $A(.)$ represents the Arrow-Pratt coefficient of absolute risk-aversion, which is assumed to be decreasing in final wealth. This assumption is equivalent to that made by Yitzhaki (1974). Under RDEU axiomatics, risk aversion is not given by the Arrow-Pratt coefficient (see e.g. Courtault and Gayant, 1998). One may then establish the following COROLLARY to PROPOSITION 2 involves a comparison of the magnitude of the comparative statics under the two axiomatics:

Corollary 1 ***(comparative statics):*** *For* $p < \underline{p}^*_{RDEU}(\theta)$ $(z^*_{EU} > z^*_{RDEU} > 0)$, *and under* CONDITION 2, *we have:*

$$(i) \quad 0 < \frac{dz^*_{RDEU}}{dy} < \frac{dz^*_{EU}}{dy};$$

$$(ii) \quad \frac{dz^*_{RDEU}}{dt} < \frac{dz^*_{EU}}{dt} < 0.$$

PROOF: SEE APPENDIX.

The direction of the comparative statics results established in COROLLARY 1 have been the subject of much debate in the tax compliance literature (see e.g. Andreoni *et al*, 1998). For example, while Feinstein (1991) presents empirical results that correspond to the theoretical predictions of EU and RDEU models, Clotefelter (1983) (empirically) and Friedland, Maital and Rutenberg (1978) or Alm, Jackson and McKee (1992) (experimentally) obtain the opposite, i.e. $\frac{dz^*}{dt} > 0$. Be this as it may, the signs of the comparative statics in the Yitzhaki (1974) formulation are similar under EU and RDEU axiomatics, though COROLLARY 1 shows that quantitative differences can arise.

Proposition 2 is illustrated in Figure 1, which uses the single parameter probability weighting function proposed by Tversky and Kahneman (1992),

$$\varphi(p) = \frac{p^\gamma}{\left[p^\gamma + (1-p)^\gamma\right]^{\frac{1}{\gamma}}},$$

specified by Abdellaoui (2000) with $\gamma = 0.7$, with a CRRA utility function ($u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, $\sigma = 1,8$), and $\theta = 2$, $t = 0.30$.

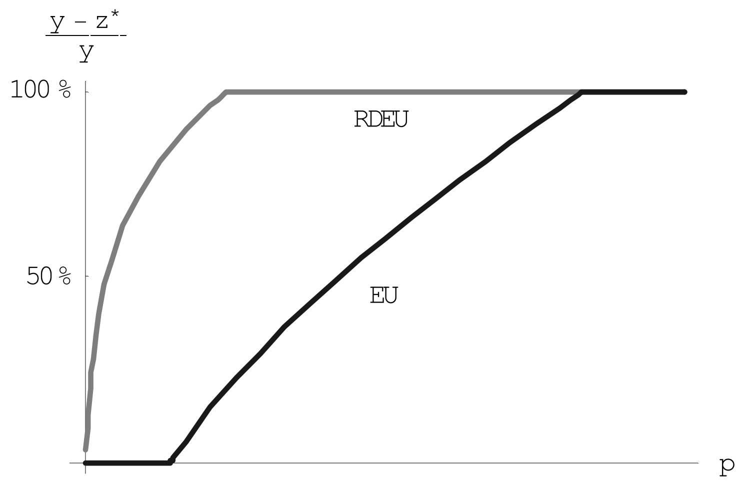

Figure 1: Compliance rate ($\frac{y-z^*_{EU}}{y}$ and $\frac{y-z^*_{RDEU}}{y}$) as a function of the audit probability (p).

Full compliance obtains when the compliance rate curves reach the 100% level. As should be obvious from Figure 1, this occurs for much lower values of p under RDEU axiomatics than in the EU case.

3 Numerical simulations

Proposition 2 establishes a relationship between the audit probability and the penalty rate that ensures full compliance, expressed in terms of the threshold audit probabilities:

$$\overline{p}^*_{EU}(\theta) = \frac{1}{1+\theta} \quad \text{and} \quad \overline{p}^*_{RDEU}(\theta) = 1 - \varphi^{-1}\left(\frac{\theta}{1+\theta}\right) \tag{9}$$

Now note that:

$$\frac{\partial\left(\overline{p}^{*}_{EU}(\theta)-\overline{p}^{*}_{RDEU}(\theta)\right)}{\partial\theta}=-\frac{1}{(1+\theta)^{2}}\left[1+\varphi^{-1'}\left(\frac{\theta}{1+\theta}\right)\right]<0.$$

The difference between $\overline{p}^{*}_{EU}(\theta)$ and $\overline{p}^{*}_{RDEU}(\theta)$ is thus a decreasing function of the penalty rate θ. This statement can be formulated in an alternative manner by seeking to determine the minimal penalty rate that entails full compliance, for a given value of the probability of audit p. Formally-speaking, these "limit" penalty rates can be expressed as:

$$\theta^{*}_{EU}=\frac{1-p}{p}\quad\text{and}\quad\theta^{*}_{RDEU}=\frac{\varphi(1-p)}{1-\varphi(1-p)}.$$

Since $\varphi(1-p)<1-p$, it is immediate that $\theta^{*}_{EU}>\theta^{*}_{RDEU}$. It follows that when the penalty rate θ belongs to the interval $[\theta^{*}_{RDEU},\theta^{*}_{EU})$, RDEU axiomatics predict full compliance, as opposed to the EU model under which some cheating will obtain. Generalizing the Allingham-Sandmo-Yitzhaki model to RDEU axiomatics therefore strengthens the deterrence effect of the penalty rate.

In what follows, we shall consider the following parameterizations of the probability weighting function:[6]

Authors	**Probability Weighting Function**: $\varphi(.)$
Camerer and Ho (1994)	$\frac{p^{\beta}}{\left(p^{\beta}+(1-p)^{\beta}\right)^{\frac{1}{\beta}}},\quad \beta=0.56$
Tversky and Fox (1995)	$\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}},\quad \delta=0.77$ and $\gamma=0.69$
Prelec (1998)	$\exp\left[-\left(-\ln[p]\right)^{\alpha}\right],\quad \alpha=0.4$

Bernasconi (1998) uses the Camerer and Ho (1994) probability weighting function in his work on tax avoidance. It is clear at this stage that experimental work, in the specific context

[6] While some of these probability weighting functions were developed in a CPT context, they are readily transposed to the RDEU approach (see e.g. Camerer, 1994). Our simulations, carried out Mathematica 5.0 are available upon request.

of tax-avoidance, would be extremely useful in setting the parameter values correctly in the simulations that follow. Given the lack of such experimental evidence, our results should be taken with a grain of salt, although they are likely to be representative of the broad differences between the two parameterizations. The following Table reports the penalty rate needed to ensure full compliance, as a function of the audit probability, for the following specifications.[7]

p	EU	Bernasconi	Tversky	Abdellaoui	Prelec
1.0%	99.00	7.39	18.34	16.65	5.81
1.5%	65.66	5.00	13.82	12.75	4.86
2.0%	49.00	3.97	11.29	10.54	4.28
2.5%	39.00	3.36	9.65	9.10	3.87
3.0%	32.33	2.96	8.47	8.05	3.56
3.5%	27.57	2.66	7.59	7.25	3.32
4.0%	24.00	2.43	6.90	6.63	3.12
4.5%	21.22	2.24	6.34	6.12	2.95
5.0%	19.00	2.09	5.87	5.69	2.81

Table 1: Critical penalty rates

The first column in Table 1 corresponds to the audit probabilities. The results presented in Table 1 do not depend upon the specification chosen for the utility function. For this reason, we find the Abdellaoui (2000) specification (where the probability weighting function is the same as in Tversky and Fox, with $\delta = 0.84$ and $\gamma = 0.65$ for losses) particularly interesting as it is not based on an appeal to a particular functional form for the utility function.

Using the Prelec (1998) specification, Table 2 presents the value of the penalty rate that

[7] The "Bernasconi" acronym corresponds to Bernasconi (1998) who used the Camerer and Ho (1994) specification, while "Tversky" corresponds to Tversky and Fox (1995), "Abdellaoui" to Abdellaoui (2000) and "Prelec" to Prelec (1998).

ensures full compliance, as a function of the parameter α; for comparison purposes, we also present the corresponding critical penalty rates under EU axiomatics.[8]

		Prelec probability weighting function				
p	EU	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.8$
1.0%	99.00	2.042	3.496	9.483	15.306	39.15
1.5%	65.66	1.848	3.040	7.644	11.877	28.11
2.0%	49.00	1.720	2.749	6.547	9.901	22.18
2.5%	39.00	1.625	2.540	5.798	8.586	18.43
3.0%	32.33	1.551	2.379	5.244	7.634	15.83
3.5%	27.57	1.490	2.249	4.814	6.906	13.91
4.0%	24.00	1.439	2.142	4.466	6.327	12.42
4.5%	21.22	1.395	2.050	4.178	5.853	11.24
5.0%	19.00	1.357	1.971	3.934	5.456	10.27

Table 2: Critical penalty rates for the Prelec specification

As should be clear from Table 2, the Prelec (1998) specification with $\alpha = 0.3$ yields a significant overestimation of low audit probabilities, with full compliance obtaining for an audit probability of 2% and a penalty rate of 2.75. For this audit probability, the corresponding threshold penalty rate is equal to 49.0 under EU axiomatics.

An additional exercise involves simulating the rate of compliance $(1 - z/y)$ for "realistic" values of the audit probability and the penalty rate. If we consider the parameterization chosen by Bernasconi (1998), which involves a CRRA utility function $u(x) = \frac{x}{1-\sigma}^{1-\sigma}$ with

[8] As in Table 1, the first column corresponds to various audit probabilities (p).

$\sigma = 1.8$, t equal to 0.3, and $\theta = 3$, we obtain the simulation results presented in Table 3.

p	EU	Bernasconi	Tversky	Abdellaoui	Prelec ($\alpha = 0.4$)
1.0%	0.091	0.745	0.371	0.395	0.722
1.5%	0.140	0.802	0.443	0.465	0.791
2.0%	0.182	0.847	0.500	0.521	0.843
2.5%	0.218	0.883	0.548	0.567	0.886
3.0%	0.252	0.913	0.589	0.606	0.922
3.5%	0.282	0.940	0.626	0.642	0.954
4.0%	0.311	0.964	0.659	0.674	0.982

Table 3: Rates of Compliance, $t = 0.3, \theta = 3$ and $u(x) = \dfrac{x^{1-\sigma}}{1-\sigma}$ with $\sigma = 1.8$.

Here, with $\alpha = 0.3$, full compliance obtains for audit probabilities greater than 2%;

4 Concluding remarks

Considering the Allingham-Sandmo-Yitzhaki problem under RDEU axiomatics has allowed us to bridge at least part of the gap between observed levels of compliance and theoretical predictions. Intuitively, RDEU axiomatics allow one to do this by introducing "pessimism" into the individual's decision-making process in that the taxpayer will overestimate the probability of audit. Several extensions to our approach could be envisaged, such as integrating occupational choice or labor supply. Contrary to Bernasconi (1998), we believe that social or ethical factors may still constitute an important portion of the tax compliance puzzle, insofar as they affect the probability weighting function, although we have not developed this point here. Experimental evidence would be extremely useful in this context. An example includes the concept of "competence" as defined by Heath and Tversky (1991), which explains the use of accountants for establishing tax returns. Another is the model of Pestieau, Possen and Slustky (1998) who consider, in an EU context, a lottery in terms of the tax code. A similar lottery under Non EU axiomatics would allow one to distinguish individuals according to their aversion to ambiguity, as defined by Ellsberg (1961).

5 Appendix: Proof of Corollary 1

We begin by rewriting the FOCs under the two axiomatics (EU and RDEU), where one denotes $\widetilde{y}$ explicitly by $y(1-t)$ and where we restrict our attention to situations where $p < \underline{p}^*_{RDEU}(\theta)$:

$$t\left[(1-p)\,u'\left(y(1-t)+tz^*_{EU}\right)-\theta p u'\left(y(1-t)-\theta t z^*_{EU}\right)\right]=0, \tag{10}$$

$$t\begin{bmatrix} \varphi(1-p)\,u'\left(y(1-t)+tz^*_{RDEU}\right) \\ -\theta\left[1-\varphi(1-p)\right]u'\left(y(1-t)-\theta t z^*_{RDEU}\right)\end{bmatrix}=0. \tag{11}$$

An application of the Implicit Function Theorem to (10) then yields:

$$\frac{dz^*_{EU}}{dy}=-\frac{(1-t)\left[(1-p)\,u''\left(w^N_{EU}\right)-\theta p u''\left(w^A_{EU}\right)\right]}{t\left[(1-p)\,u''\left(w^N_{EU}\right)+\theta^2 p u''\left(w^A_{EU}\right)\right]}.$$

In order to lighten notation, let w^N be the taxpayer's income when she is not audited ($w^N = \widetilde{y}+tz$), whereas w^A corresponds to her income when she is ($w^N = \widetilde{y}-\theta tz$). The subscript corresponds to the axiomatics being used. Moreover, we define $A(.) = -\frac{u''(.)}{u'(.)}$ as the "sensitivity" of marginal utility, where we assume Condition 2 respected.

From the FOCs, we know that:

$$u'\left(w^A_{EU}\right)=\frac{1-p}{\theta p}u'\left(w^N_{EU}\right), \tag{12}$$

and

$$u'\left(w^A_{RDEU}\right)=\frac{\varphi(1-p)}{\theta\left[1-\varphi(1-p)\right]}u'\left(w^N_{RDEU}\right). \tag{13}$$

Substituting for $A(.)$, combining this with (12) and simplifying by $(1-p)$ and $u'\left(w_{EU}^{N}\right)$, then yields:

$$\frac{dz_{EU}^{*}}{dy}=\frac{(1-t)\left[A\left(w_{EU}^{A}\right)-A\left(w_{EU}^{N}\right)\right]}{t\left[A\left(w_{EU}^{N}\right)+\theta A\left(w_{EU}^{A}\right)\right]}.$$

Since $A(.)$ is assumed to be decreasing (CONDITION 2), it follows, since $w_{EU}^{A}<w_{EU}^{N}$, that $A\left(w_{EU}^{A}\right)>A\left(w_{EU}^{N}\right)$. The consequence, since the denominator is the SOC and is therefore negative, is that $\frac{dz_{EU}^{*}}{dy}>0$. Similarly, one can compute:

$$\frac{dz_{RDEU}^{*}}{dy}=\frac{(1-t)\left[A\left(w_{RDEU}^{A}\right)-A\left(w_{RDEU}^{N}\right)\right]}{t\left[A\left(w_{RDEU}^{N}\right)+\theta A\left(w_{RDEU}^{A}\right)\right]}>0.$$

Our purpose here is to prove part (i) of the COROLLARY, namely that $\frac{dz_{RDEU}^{*}}{dy}<\frac{dz_{EU}^{*}}{dy}$. An important consequence of part (i) of PROPOSITION 1 is that:

$$w_{EU}^{N}>w_{RDEU}^{N}>w_{RDEU}^{A}>w_{EU}^{A}.$$

Since $A(.)$ is decreasing, it follows that

$$A\left(w_{EU}^{N}\right)<A\left(w_{RDEU}^{N}\right)<A\left(w_{RDEU}^{A}\right)<A\left(w_{EU}^{A}\right), \tag{14}$$

and therefore

$$A\left(w_{EU}^{N}\right)A\left(w_{RDEU}^{A}\right)<A\left(w_{RDEU}^{N}\right)A\left(w_{EU}^{A}\right),$$

which implies that

$$\frac{dz_{RDEU}^{*}}{dy}<\frac{dz_{EU}^{*}}{dy}.$$

This proves part (i) of COROLLARY 1. For part (ii) of COROLLARY 1, implicit differentiation of the FOC (10) and use of (12) and (14) yields:

$$\frac{dz_{EU}^{*}}{dt}=-\frac{y\left[A\left(w_{EU}^{A}\right)-A\left(w_{EU}^{N}\right)\right]+z_{EU}^{*}\left[\theta A\left(w_{EU}^{A}\right)+A\left(w_{EU}^{N}\right)\right]}{t\left[A\left(w_{EU}^{N}\right)+\theta A\left(w_{EU}^{A}\right)\right]}<0.$$

In the case of RDEU, similar computations yield:

$$\frac{dz^*_{RDEU}}{dt} = -\frac{y\left[A\left(w^A_{RDEU}\right) - A\left(w^N_{RDEU}\right)\right] + z^*_{RDEU}\left[\theta A\left(w^A_{RDEU}\right) + A\left(w^N_{RDEU}\right)\right]}{t\left[A\left(w^N_{RDEU}\right) + \theta A\left(w^A_{RDEU}\right)\right]} < 0.$$

The next step is to compare $\frac{dz^*_{EU}}{dt}$ and $\frac{dz^*_{RDEU}}{dt}$, where we can write:

$$\begin{aligned}\frac{dz^*_{RDEU}}{dt} &= \frac{dz^*_{EU}}{dt}\left(\frac{A\left(w^N_{EU}\right) + \theta A\left(w^A_{EU}\right)}{A\left(w^N_{RDEU}\right) + \theta A\left(w^A_{RDEU}\right)}\right) \\ &\quad \left(\frac{y\left[A\left(w^A_{RDEU}\right) - A\left(w^N_{RDEU}\right)\right] + z^*_{RDEU}\left[\theta A\left(w^A_{RDEU}\right) + A\left(w^N_{RDEU}\right)\right]}{y\left[A\left(w^A_{EU}\right) - A\left(w^N_{EU}\right)\right] + z^*_{EU}\left[\theta A\left(w^A_{EU}\right) + A\left(w^N_{EU}\right)\right]}\right).\end{aligned}$$

We will now prove that:

$$\left(\frac{A\left(w^N_{EU}\right)+\theta A\left(w^A_{EU}\right)}{A\left(w^N_{RDEU}\right)+\theta A\left(w^A_{RDEU}\right)}\right)\left(\frac{y\left[A\left(w^A_{RDEU}\right)-A\left(w^N_{RDEU}\right)\right]+z^*_{RDEU}\left[\theta A\left(w^A_{RDEU}\right)+A\left(w^N_{RDEU}\right)\right]}{y\left[A\left(w^A_{EU}\right)-A\left(w^N_{EU}\right)\right]+z^*_{EU}\left[\theta A\left(w^A_{EU}\right)+A\left(w^N_{EU}\right)\right]}\right) < 1.$$

Note that the preceding inequality is equivalent to:

$$\begin{aligned} &\left(A\left(w^N_{EU}\right) + \theta A\left(w^A_{EU}\right)\right)\left(\left(y + \theta z^*_{RDEU}\right) A\left(w^A_{RDEU}\right) - \left(y + z^*_{RDEU}\right) A\left(w^N_{RDEU}\right)\right) \\ < &\left(A\left(w^N_{RDEU}\right) + \theta A\left(w^A_{RDEU}\right)\right)\left(\left(y + \theta z^*_{RDEU}\right) A\left(w^A_{EU}\right) - \left(y + z^*_{RDEU}\right) A\left(w^N_{EU}\right)\right),\end{aligned}$$

or

$$\begin{aligned} &\left(y + \theta z^*_{RDEU}\right) A\left(w^N_{EU}\right) A\left(w^A_{RDEU}\right) - \left(y + z^*_{RDEU}\right) A\left(w^N_{EU}\right) A\left(w^N_{RDEU}\right) \\ &+\theta\left(y + \theta z^*_{RDEU}\right) A\left(w^A_{EU}\right) A\left(w^A_{RDEU}\right) - \theta\left(y + z^*_{RDEU}\right) A\left(w^A_{EU}\right) A\left(w^N_{RDEU}\right) \\ < &\left(y + \theta z^*_{RDEU}\right) A\left(w^A_{EU}\right) A\left(w^N_{RDEU}\right) + \theta\left(y + \theta z^*_{RDEU}\right) A\left(w^A_{EU}\right) A\left(w^A_{RDEU}\right) \\ &-\left(y + z^*_{RDEU}\right) A\left(w^N_{EU}\right) A\left(w^N_{RDEU}\right) - \left(y + z^*_{RDEU}\right)\theta A\left(w^N_{EU}\right) A\left(w^A_{RDEU}\right).\end{aligned}$$

Cancelling out terms then implies that this boils down to:

$$A\left(w^N_{EU}\right) A\left(w^A_{RDEU}\right) < A\left(w^A_{EU}\right) A\left(w^N_{RDEU}\right),$$

whence,

$$\frac{dz^*_{RDEU}}{dt} < \frac{dz^*_{EU}}{dt} \ [QED] \, .$$

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