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Who Would Benefit from Simplifying the Tax Code?
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Abstract

We consider the classic Allingham and Sandmo (1972) tax compliance problem in the context of the Choquet-Schmeidler Expected Utility (CSEU) model, using the Non–Extremal Outcome (NEO)-additive capacities proposed by Chateauneuf, Eichberger, and Grant (2004), in which Knightian uncertainty (ambiguity) exists concerning the penalty rate faced in the case of an audit. Pessimistic incarnations of the CSEU model can yield much lower underreporting rates than its Expected Utility (EU) counterpart, and do so without the need for moral sentiments, social stigma or probability perception functions. We confirm previous results, obtained in other contexts, showing that ambiguity-aversion reinforces the incentive effects of risk-aversion. We define the concept of a Risk-preserving increase in ambiguity (RPIA), which allows us to consider a change in the distribution of penalty rates such that (i) a CSEU decisionmaker will perceive a change in her welfare, whereas (ii) an EU decisionmaker will not. We also present simulation results that support the view according to which ambiguity aversion explains the use of accounting firms in preparing tax returns. Finally, by modeling a simple game between the taxpayer and the Internal Revenue Service (IRS), we show that increasing ambiguity in the tax code will not be in the IRS's interest if the associated rise in the cost of auditing is sufficiently large. It is therefore likely that increasing complexity (and therefore ambiguity) will *reduce* tax receipts, even in the presence of ambiguity-averse taxpayers.

Keywords: Choquet expected utility, ambiguity, tax compliance

JEL: D81, H26.

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1 Introduction

Recent work by Bernasconi (1998), Bernasconi and Zanardi (2004) and Arcand and Rota-Graziosi (2004) has shown that pessimism concerning audit rates, formalized using the Rank Dependent Expected Utility (RDEU) model or Cumulative Prospect Theory (CPT), can explain the abnormally high tax compliance rates observed in practice, abnormal, that is, in terms of an Expected Utility (EU) interpretation (see Andreoni, Erard, and Feinstein (1998) for a survey).[1] The purpose of this paper, in contrast, is threefold.

First, we extend the standard Allingham and Sandmo (1972) model of tax compliance as a gamble to situations of *ambiguity*, also known as *Knightian uncertainty*. This will allow us to ascertain the degree to which *ambiguity aversion* reinforces risk aversion, and can therefore contribute something to explaining the tax compliance puzzle. That ambiguity aversion reinforces risk aversion has been shown in other contexts (see Mukerji and Tallon (2004) for a survey).

Second, we highlight the use of professional accountants in preparing tax returns, where their role is interpreted as being one of eliminating or at least reducing the ambiguity associated with the tax compliance gamble. By making use of an accountant, the taxpayer essentially tilts the world towards one characterized by risk, rather than one characterized by uncertainty. In particular, we compute the willingness to pay for eliminating ambiguity, and distinguish between the "pure" ambiguity premium and the "aggregate" premium which includes the effect of risk.

Third, we go beyond the basic Allingham and Sandmo (1972) framework, and consider an extremely simple game in which the tax authorities choose an audit probability and the taxpayer her degree of compliance (Graetz, Reinganum, and Wilde (1986)). The properties of the equilibrium under ambiguity (EUA) of this game imply that, when the decrease in

[1] In the EU model, strictly positive underreporting will obtain when the expected gain to evasion (the probability of not being audited *minus* the probability of being audited *times* the penalty rate one incurs on undeclared income) is positive: in most countries, this expected gain is indeed positive, but most taxpayers engage in no underreporting at all.

the cost of an audit caused by simplification of the tax code (and thus the elimination of ambiguity) is sufficiently large, it will be in the interest of the tax authorities to eliminate ambiguity in the tax code, despite its deterrence effect in terms of taxpayer compliance in a partial equilibrium setting. All of our theoretical results are, when possible, confronted with empirical evidence for the US.

Our focus on the ambiguity of the tax code is motivated empirically by the important public policy implications of the issue. For example, the Cato Institute states that:

> Income taxes are hard to understand and the rules now span 60,044 pages.... Americans are baffled by the complex rules on capital gains, savings plans, education incentives, and other items.... Tax complexity is getting worse.... Citizens are required to comply with the tax laws, but that is difficult when the rules are constantly changing.

Few would argue with the statement that tax codes, in the US and other OECD countries, are unduly complex, and in the US case, the issue was recently brought to the forefront of public debate by the widely publicized report of the National Taxpayer Advocate (2004):[2]

> Without a doubt, the largest source of compliance burdens for taxpayers and the IRS alike is the overwhelming complexity of the tax code, and without a doubt, the only meaningful way to reduce these compliance burdens is to simplify the tax code enormously.

Given this state of affairs, is it reasonable to assume, first, that taxpayers know the penalty rates that apply in different circumstances when tax is underreported and, second, that taxpayers are able to assign probabilities to the various potential outcomes they may be confronted with?

We pose these two questions at the outset because they are key assumptions of the standard EU approach to tax compliance. We believe that both assumptions are unreasonable, and that the EU approach therefore ignores important aspects of taxpayer behavior. The alternative proposed in this paper is therefore to place the ambiguity of the tax code, generated by its complexity, at the heart of the analysis, using the Choquet-Schmeidler Expected

[2] The full report is available online at: http://www.irs.gov/advocate/article/0,,id=133967,00.html

Utility (CSEU) model developed by Schmeidler (1989). We also choose to focus on uncertainty concerning the penalty rate, though an alternative would be to focus on uncertainty concerning the marginal tax rate.[3]

The structure of the remainder of the paper is as follows. In part 2, we present the assumptions under which we shall be working, and specify preferences in terms of the CSEU model with NEO-additive capacities recently proposed by Chateauneuf, Eichberger, and Grant (2004). In part 3 we specify the tax-compliance gamble in the context of CSEU preferences with NEO-additive capacities, with an uncertain penalty rate, and characterize optimal compliance behavior (Proposition 1). In part 4 we study the impact of changes in the distribution of the penalty rate on compliance behavior. The changes in the distribution of penalty rates we consider are all meant as proxies for a change in the complexity of the tax code. In turn, we consider: (i) a mean preserving increase in the risk of the distribution of penalty rates (Proposition 2), (ii) an $\alpha-$squeeze in the distribution of penalty rates (Proposition 4), and we introduce (iii) the concept of a "risk preserving increase in the ambiguity" of the distribution of penalty rates (Proposition 5). In passing, we verify that the results obtained in the CSEU context are robust (Proposition 3) to a switch to alternative axiomatics provided by the "smooth model of ambiguity aversion" of Klibanoff, Marinacci, and Mukerji (2004). All of these results confirm earlier findings (summarized in Mukerji and Tallon (2004)) that ambiguity-aversion (when pessimism prevails) tends to reinforce the effects of risk-aversion. We also provide empirical evidence which suggests that a pessimistic CSEU model with NEO-additive capacities provides a good explanation for observed patterns of underreporting in the US over the past half century. In part 5, we provide a general characterization of the risk and ambiguity premia in the CSEU with NEO-additive capacities case (Lemma 3), and apply the result to the tax-compliance context (Proposition 5). Finally, in part 6, we specify the interaction between the taxpayer and the IRS in a simple game-theoretic construct (in which the taxpayer chooses her level of

[3] Our paper also extends the results obtained in an EU setting by Alm, Jackson, and McKee (1992b) to a CSEU setting.

underreporting and the IRS chooses the audit probability), and analyze the impact on the resulting equilibrium of changes in the distribution of penalty rates. Of particular interest in this context is how changes in the distribution of penalty rates affect the cost of an audit for the IRS. We prove (Proposition 6) that an increase in complexity (as proxied by a mean preserving increase in the risk of penalty rates) will *increase* equilibrium underreporting when the resulting increase in the cost of an audit is sufficient to offset the deterrence effect on taxpayers of increased complexity.

2 Capacities and Choquet-Schmeidler expected utility

In this paper, it will be convenient to focus our attention on Non-Extremal Outcome additive capacities (henceforth "NEO-additive") defined by Chateauneuf, Eichberger, and Grant (2004). Following these authors, we shall place ourselves in a situation of *uncertainty*: a state of nature will obtain, but we will be unable to say definitely which one. Let $S = \{s_0, s_1, ..., s_i, ..., s_n\}$ be the finite set of states of nature. Consider the set of subsets of S, denoted by $\mathbf{E} = 2^S$, which we shall refer to as the set of *events*. Let $X : S \to \mathbb{R}$ with $s \to X(s)$. Then a capacity is defined as follows:

Definition 1 $\nu : A \in \mathbf{E} \to \nu(A) \in [0,1]$ *is a capacity if* $\nu(\varnothing) = 0$, $\nu(S) = 1$, *and* $A \subseteq B \Rightarrow \nu(A) \leqslant \nu(B)$; ν *is convex if* $\nu(A \cup B) + \nu(A \cap B) \geqslant \nu(A) + \nu(B), \forall A, B \in \mathbf{E}$.

Definition 2 *(Chateauneuf, Eichberger, and Grant (2004), Definition 3.1) For all* $E \in \mathbf{E}$,

$$\mu^0(E) \equiv \begin{cases} 1 & for \quad E = S \\ 0 & otherwise \end{cases}, \quad \mu^1(E) \equiv \begin{cases} 1 & for \quad E = \emptyset \\ 0 & otherwise \end{cases}.$$

Let π be a probability distribution defined over $\mathbf{E}$. If we assume that π is objectively given, then the CSEU model with Neo-additive capacity is equivalent to the model of Cohen (1992) under risk. We use the following definitions:

Definition 3 *(Chateauneuf, Eichberger, and Grant (2004), Definition 3.2) Let* γ, λ *be real numbers such that* $0 \leqslant \gamma \leqslant 1$, $0 \leqslant \lambda \leqslant 1 - \gamma$. *Then a NEO-additive capacity* ν *is defined by* $\nu(E\,|\pi, \gamma, \lambda) = \gamma\mu^0(E) + \lambda\mu^1(E) + (1 - \lambda - \gamma)\pi(E)$, $\forall E \in \mathbf{E}$.

Let $X \in V$ where $V = \{X : S \to \mathbb{R}\}$. The Choquet-Schmeidler Expected Utility (CSEU) is defined as follows:

Definition 4 *(Choquet (1953), Schmeidler (1986)) Let $X = x_0 A_0^* + ... + x_i A_i^* + ... x_n A_n^*$, where A_i is a partition of S and A_i^* is an indicator function defined by*

$$A_i^*(s) = \begin{cases} 1 \text{ if } s \in A \\ 0 \text{ otherwise} \end{cases}$$

and where we rank $x_n \leqslant ... \leqslant x_i \leqslant ... \leqslant x_1 \leqslant x_0$. Then the CSEU of the gamble is given by $\int u(X) d\nu = u(x_n) + (u(x_{n-1}) - u(x_n))\, \nu(X \geqslant x_{n-1}) + ... + (u(x_{i-1}) - u(x_i))\, \nu(X \geqslant x_{i-1}) + ... + (u(x_0) - u(x_1))\, \nu(X \geqslant x_0)$.

In the case of NEO-additive capacities, the expectation of $u(.)$ with respect to the NEO-additive capacity $\nu(.)$, is also given by the Choquet integral, which takes a particularly intuitive form:

Lemma 1 *(Chateauneuf, Eichberger, and Grant (2004), Lemma 3.1) The CSEU of a simple function $f : S \longrightarrow \mathbb{R}$ with respect to the NEO-additive capacity $\nu(E\,|\pi, \gamma, \lambda)$ is given by:*

$$V(f\,|\nu(E\,|\pi, \gamma, \lambda)) = \gamma \min_{s \in S} f + \lambda \max_{s \in S} f + (1 - \lambda - \gamma)\, E_\pi[f], \quad \forall E \in \mathbf{E}.$$

The sum $\gamma + \lambda$ represents the amount of *perceived* ambiguity, $1 - \gamma - \lambda$ is the degree of confidence in the belief π. In what follows we use the lack of confidence in the belief represented by π as one of the main explanatory factors for the demand for professional accountant's services.

3 Tax compliance under Choquet-Schmeidler expected utility with NEO-additive capacities

Consider the standard Allingham and Sandmo (1972) tax compliance problem, where we allow for different potential penalty rates, denoted by $\theta_i, i = 1, ..., n$, with $\theta_1 < \theta_2 < ... < \theta_i < \theta_{i+1} < ... < \theta_n$. Let y denote after-tax income, t the marginal tax rate and z the extent of underreporting. Then the gamble faced by the taxpayer involves monetary outcomes that can be ranked, for $z \geqslant 0$, as $y - \theta_n tz \leqslant ... \leqslant y - \theta_i tz \leqslant ... \leqslant y - \theta_2 tz \leqslant y - \theta_1 tz \leqslant y + tz$, where $y - \theta_i tz$ represents the outcome when an audit obtains and penalty rate θ_i is applied, whereas $y + tz$ represents the outcome in the absence of an audit.[4] We assume that the probability of audit is known to the taxpayer and equal to p, while the (unknown) probability of facing penalty rate θ_i in the case of an audit is equal to q_i, with $\sum_{i=1}^{i=n} q_i = 1$.

Note that the problem here is more complex than in the standard model. The taxpayer has the choice between reporting her true income and avoiding penalties (her gain in this case is known and certain) or underreporting her income and facing an audit which could lead to penalties of an uncertain magnitude. The key insight from the modelling that follows is that the taxpayer's behavior is not only driven by risk-aversion: it is driven by ambiguity-aversion as well, since the probabilities with which the different penalty rates obtain are unknown.

Applying CSEU with NEO–additive capacities to the tax compliance problem yields the following objective function for the taxpayer:

$$CSEU\left(\gamma, \lambda; z\right) = p \left[\begin{array}{c} \gamma u\left(y - \theta_n tz\right) + \lambda u\left(y - \theta_1 tz\right) \\ + \left(1 - \gamma - \lambda\right) \sum_{i=1}^{i=n} q_i u\left(y - \theta_i tz\right) \end{array} \right] + \left(1 - p\right) u(y + tz). \qquad (1)$$

When it is convenient, we shall replace the discrete specification in terms of $\sum_{i=1}^{i=n} q_i u\left(y - \theta_i tz\right)$ by a continuous formulation of the form $\int_{\theta_1}^{\theta_n} u\left(y - \theta tz\right) q(\theta) d\theta$, where $q(\theta)$ denotes the probability density function (*p.d.f.*) according to which θ is distributed. In terms of optimal compliance behavior, one can then readily establish the following result:

Proposition 1 *Let* μ_θ *denote the mean penalty rate,* σ_θ^2 *its variance, and let* $A\left(y\right) = -\frac{u''(y)}{u'(y)}$. *Then a second-order approximation to optimal underreporting is given by* $z^*_{CSEU}\left(\gamma, \lambda\right) = \frac{1}{tA(y)} \frac{1-p-p\left(\theta_n\gamma+\theta_1\lambda+(1-\gamma-\lambda)\mu_\theta\right)}{1-p+p\left(\theta_n^2\gamma+\theta_1^2\lambda+(1-\gamma-\lambda)\left(\mu_\theta^2+\sigma_\theta^2\right)\right)}$, *when* $p < p_{CSEU}\left(\gamma, \lambda\right) = \frac{1}{1+\theta_n\gamma+\theta_1\lambda+(1-\gamma-\lambda)\mu_\theta}$, *and* $z^*_{CSEU}\left(\gamma, \lambda\right) = 0$, *otherwise.*

Proof: See Appendix.

Optimal compliance behavior under CSEU with NEO–additive capacities yields several well-known special cases (see Chateauneuf, Eichberger, and Grant (2004)) by restricting

[4] This formulation of the problem, in which the total penalty is proportional to the amount evaded, is due to a classic paper by Yitzhaki (1974).

the values taken on by γ and λ.[5] For $\gamma = \lambda = 0$ we obtain the EU solution $z^*_{EU} = z^*_{CSEU}(0,0) = \frac{1}{tA(y)} \frac{1-p-p\mu_\theta}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)}$.[6] The restriction $0 < \gamma \leqslant 1, \lambda = 0$ yields the *simple capacity* case, whose axiomatics have been provided by Eichberger and Kelsey (1999): $z^*_{CSEU}(\gamma, 0) = \frac{1}{tA(y)} \frac{1-p-p\mu_\theta - p\gamma(\theta_n - \mu_\theta)}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right)}$.[7] Gajdos, Tallon, and Vergnaud (2004) extend the Gilboa-Schmeidler maxmin expected utility model, yielding a specification that is functionally equivalent to this case, where the parameter γ is interpreted as being the degree of "ambiguity-aversion" of the decisionmaker (also see Mukerji (1997)). When $\gamma = 1, \lambda = 0$, which corresponds to a situation called "pure pessimism", $z^*_{CSEU}(1,0) = \frac{1}{tA(y)} \frac{1-p-p\theta_n}{1-p+p\theta_n^2}$. Conversely, optimism corresponds to $0 < \lambda \leqslant 1, \gamma = 0$ and $z^*_{CSEU}(0, \lambda) = \frac{1}{tA(y)} \frac{1-p-p\mu_\theta - p\lambda(\theta_1-\mu_\theta)}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)}$. Finally, when $\gamma + \lambda = 1$, one obtains $z^*_{CSEU}(\gamma, 1-\gamma) = \frac{1}{tA(y)} \frac{1-p-p(\theta_1+\gamma(\theta_n-\theta_1))}{1-p+p\left(\theta_1^2+\gamma\left(\theta_n^2-\theta_1^2\right)\right)}$.

Let p_{EU} denote the threshold value of the audit probability below which positive underreporting obtains in the EU case. Then Proposition 1 implies that $p_{CSEU}(\gamma, \lambda) < p_{EU}$ when

$$\frac{\gamma}{\lambda} > \frac{\mu_\theta - \theta_1}{\theta_n - \mu_\theta}. \tag{2}$$

Similarly, Proposition 1 implies that $z^*_{CSEU} \leqslant z^*_{EU}$ when

$$\frac{\gamma}{\lambda} > \frac{(1-p)\left[\mu_\theta\left(1+\mu_\theta+\mu_\theta^2+\sigma_\theta^2\right)+\sigma_\theta^2\right]-\left(1-p+\mu_\theta^2+\sigma_\theta^2\right)-(1-p-p\mu_\theta)\theta_1^2}{(1-p-p\mu_\theta)\theta_n^2+\left(1-p+\mu_\theta^2+\sigma_\theta^2\right)-(1-p)\left[\mu_\theta\left(1+\mu_\theta+\mu_\theta^2+\sigma_\theta^2\right)+\sigma_\theta^2\right]}. \tag{3}$$

These inequalities are suggestive of how pessimistic CSEU preferences can provide an

[5] Note that we restrict our attention to the case where the audit rate p is known by the taxpayer. This situation is similar to the one considered in the Ellsberg (1961) paradox: the taxpayer knows p, but does not know the penalty rate she will face, the only restriction being that $q_i \in [0,1]$. An alternative specification would involve having the taxpayer not know p while restricting its value *a priori* to some interval, such as $p \in [0, \overline{p}]$. As an illustration, consider the "pessimistic" case where $\lambda = 0$ and $\gamma = 1$. Then $CSEU(1,0) = \overline{p}u(y - \theta_n tz) + (1-\overline{p})\, u(y+tz)$. An example of reasonable prior beliefs would simply be that the probability of being audited is lower than the probability of not being audited ($p \leqslant 1-p$), leading to $\overline{p} = \frac{1}{2}$ and $CSEU(1,0) = \frac{1}{2}u(y-\theta_n tz) + \frac{1}{2}u(y+tz)$. In this case, the taxpayer not only restricts her attention to the highest possible penalty rate: she also attributes a weight of $\frac{1}{2}$ to the likelihood of being audited and paying penalty rate θ_n. If $\overline{p} > \frac{1}{2}$, it is trivial to show that the taxpayer always chooses $z^* = 0$.

[6] In the standard case, $\sigma_\theta^2 = 0$ and $\mu_\theta = \theta$.

[7] Chateauneuf, Eichberger, and Grant (2004) refer to this case as being one of "pure pessimism", and do not consider the case where $\gamma = 1$ and $\lambda = 0$. Dow and Werlang (1992) and Dow and Werlang (1994) also explicitly consider the simple capacity case in which $\lambda = 1 - \gamma$.

explanation for the tax compliance puzzle: if the ratio γ/λ is sufficiently large, a CSEU taxpayer will engage in no underreporting at all for lower audit probabilities than her EU counterpart, and when they both engage in positive underreporting, the CSEU taxpayer underreports less.

In order to illustrate Proposition 1 in quantitative terms that are reasonably close to reality, consider the three-point distribution of penalty rates given by $(q_1, \theta_1;\ 1 - q_1 - q_n, \mu_\theta;\ q_n, \theta_n)$.[8] A plausible parameterization that corresponds roughly to US data involves $\theta_1 = 0$ with a probability of 0.5, and a mean penalty rate of around 20-25%, with the three probabilities satisfying $q_n \ll 1 - q_1 - q_n < q_1$. In Figure 1, underreporting as a fraction of income $(\frac{z^*_{CSEU}}{y})$ is simulated for this distribution of penalty rates, for various values of λ and γ, as a function of p. As should be clear, underreporting is much lower under CSEU preferences as long as γ is significantly greater than 0, and $z^*_{CSEU}(0.1, 0.8)$ still lies below the EU case.

[8] In this case, $\mu_\theta = \left(\frac{q_1}{q_1+q_n}\right)\theta_1 + \left(\frac{q_n}{q_1+q_n}\right)\theta_n$ and $\sigma^2_\theta = \left(\frac{q_1 q_n}{q_1+q_n}\right)(\theta_n - \theta_1)^2$. If we solve for the maximum penalty rate and its associated probability as a function of the other parameters of the distribution, we obtain $q_n = \frac{q_1^2(\theta_1-\mu_\theta)^2}{\sigma^2_\theta - q_1(\mu_\theta-\theta_1)^2}, \theta_n = \theta_1 + \frac{\sigma^2_\theta}{q_1(\mu_\theta-\theta_1)}$. Note that a maximum penalty rate (θ_n) of 2.8 greatly exceeds the maximum civil penalty rate of 25 percent of unpaid tax (75 percent in the case of fraud) that is currently used in the U.S. However, in certain situations the IRS may also initiate audits of returns from two prior tax years in addition to the current year (so-called "back audits"). Moreover, if fees paid to taxpayer representatives are included along with the potential loss of social standing, a maximum "penalty" rate of around 300 percent of the current tax year's unpaid tax could be possible in some (rare) cases.

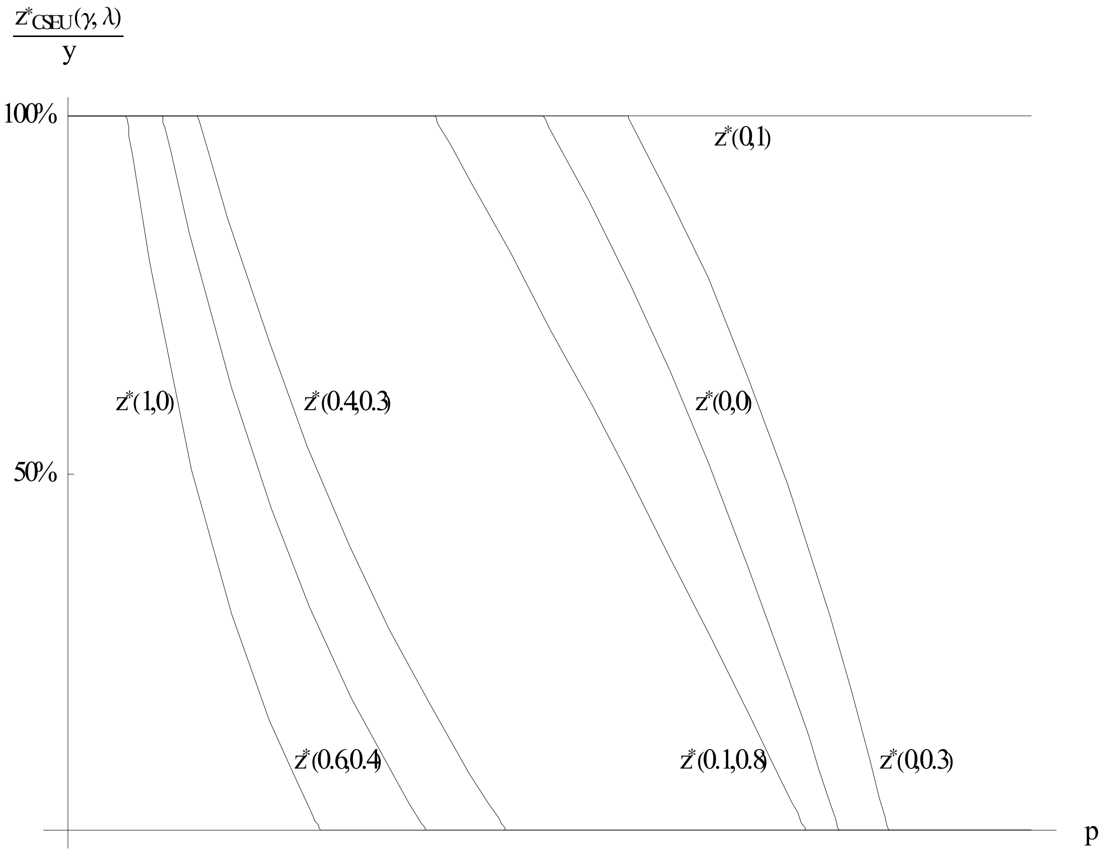

Figure 1: Optimal compliance behavior, $\frac{z^*_{CSEU}(\gamma,\lambda)}{y}$, for $u(x) = \frac{x^{1-R}}{1-R}$, $R = 1.8$, $y = 100$, $t = 0.3$, $\theta_1 = 0$, $q_1 = 0.5$, $\theta_n = 2.8$, $q_n \simeq 0.05$, $\mu_\theta = 0.25$, $\sigma^2_\theta = 0.35$.

4 Compliance behavior and changes in the distribution of penalty rates

One of the main purposes of this paper is to ascertain how changes in the ambiguity of the tax code could affect taxpayer welfare and compliance behavior. It is therefore particularly important to study the impact of changes in the distribution of penalty rates that can be interpreted as representing changes in ambiguity, as opposed to changes in γ and λ, which correspond to the *perception* of ambiguity by the decisionmaker. In what follows, we study the effects of three different changes in the distribution of penalty rates: (i) a mean-preserving increasing in its risk, (ii) a *squeeze* in the distribution of penalty rates and, (iii) a change in penalty rates that affects the extremal outcomes while leaving the moments of the distribution unchanged. Each change in the distribution of penalty rates that we consider corresponds to a different interpretation of what is meant by ambiguity *per se*, and how one chooses, or not, to distinguish it from risk.

4.1 A mean-preserving increase in risk in the distribution of θ

Gajdos, Tallon, and Vergnaud (2004) model the "degree of imprecision" in the distribution of a random variable (interpreted by them as a prior) by its risk as defined by Rothschild and Stiglitz (1970), in the context of their extension of the Gilboa-Schmeidler maxmin preference functional where $0 < \gamma \leqslant 1, \lambda = 0$.[9] The maximum degree of imprecision in their model is given by the completely uninformative (uniform) prior $q(\theta_i) = \frac{1}{n}, \forall i$. It therefore seems natural to model an increase in ambiguity by an increase in risk in the present context as well.

4.1.1 Preliminaries

Consider a Mean-Preserving Increase in the Risk (MPIR) of the distribution of penalty rates, defined by the usual integral conditions: (i) $\int_{\theta_1}^{\theta_n} Q_\rho(\theta,\rho)d\theta = 0$, (ii) $\int_{\theta_1}^{\theta'} Q_\rho(\theta,\rho)d\theta \geqslant 0, \forall\theta' \in [\theta_1, \theta_n]$, where $Q(\theta,\rho)$ is the cumulative density function (*c.d.f.*) associated with $q(\theta,\rho)$, $Q_\rho(\theta,\rho)$ is the derivative of the *c.d.f.* with respect to ρ, and ρ is the parameter of increasing risk of Rothschild and Stiglitz (1970). Other concepts of MPIRs of a distribution are considered in Chateauneuf, Cohen, and Meilijson (2004).[10]

[9] More explicitly, the Gajdos, Tallon, and Vergnaud (2004) evaluation of the tax compliance gamble would be given by $GTVEU = p\left[\gamma u(y-\theta_n tz) + (1-\gamma)\int_{\theta_1}^{\theta_n} u(y-\theta tz)q(\theta,\rho)d\theta\right] + (1-p)\,u(y+tz)$ where they would refer to γ as the taxpayer's degree of imprecision-aversion, and ρ is the Rothschild-Stiglitz parameter of increasing risk.

[10] Note, by construction, that μ_θ is independent of ρ, whereas two integrations by parts of the definition of σ_θ^2, differentiation with respect to ρ, and application of the two integral conditions yield $\frac{d\sigma_\theta^2(\rho)}{d\rho} = 2\int_{\theta_1}^{\theta_n}\left(\int_{\theta_1}^{\theta} Q_\rho(x,\rho)dx\right)d\theta \geqslant 0$. An increase in risk therefore increases variance, which will be useful in the context of the second-order approximations used in this paper, and in many of the simulations that we present. The converse is of course not necessarily true (i.e. an increase in σ_θ^2 does not necessarily

We will use two standard results associated with MPIRs, which we summarize in the following Lemma:

Lemma 2 *(Rothschild and Stiglitz (1970), Laffont (1990), Theorem 2, p. 28) Consider an expression of the form* $W(z,\rho) = \int_{\theta_1}^{\theta_n} g(\theta,z)q(\theta,\rho)d\theta$. *Then:*

$$(i)\ sign\left[\frac{d}{d\rho}W(z,\rho)\right] = sign\left[\frac{\partial^2 g(\theta,z)}{\partial\theta^2}\right];\ (ii)\ sign\left[\frac{d}{d\rho}\underset{\{z\}}{\arg\max} W(z,\rho)\right] = sign\left[\frac{\partial^3 g(\theta,z)}{\partial z\partial\theta^2}\right].$$

Part (i) of the Lemma is sometimes referred to as the *Fundamental Theorem of Risk*, whereas part (ii) often allows one to sign comparative statics unambiguously.

4.1.2 The impact of an MPIR under CSEU with NEO-additive capacities

Applying Lemma 2 to the CSEU objective function given in (1) immediately yields the following result:

Proposition 2 *Let the distribution of penalty rates be parameterized by its risk* ρ*, in the sense of Rothschild and Stiglitz (1970). Then: (i) the taxpayer's welfare, evaluated at the optimum, is decreasing in* ρ*:* $\frac{d}{d\rho}CSEU(\gamma,\lambda;z) \leqslant 0$*; (ii) optimal underreporting is also decreasing in* ρ*:* $\frac{d}{d\rho}z^*_{CSEU}(\gamma,\lambda) \leqslant 0$.

Proof: See Appendix.

Proposition 2(ii) is a general result that does not hinge upon the second-order approximation to $z^*_{CSEU}(\gamma,\lambda)$ given in Proposition 1. Note however that optimal underreporting, as approximated in Proposition 1, is of course also decreasing in ρ, because $\frac{dz^*_{CSEU}}{d\rho} = \frac{dz^*_{CSEU}}{d\sigma^2_\theta}\frac{d\sigma^2_\theta(\rho)}{d\rho} < 0$. This result establishes that, when an increase in ambiguity is modelled as an MPIR in the distribution of penalty rates, ambiguity-aversion reinforces risk-aversion by deterring underreporting.

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lead to an increase in ρ); see the counterexample in Laffont (1990), p. 26.

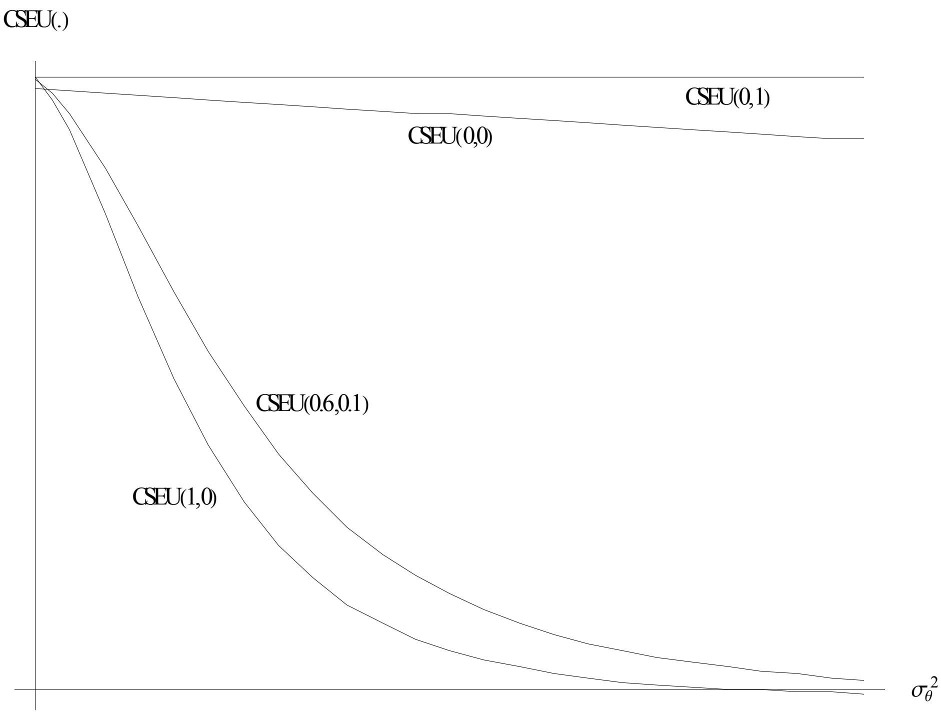

Figure 2: Welfare at the optimum under CSEU as a function of σ_θ^2, for:
$u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 100$, $t = 0.3$, $p = 0.03$, $\theta_1 = 0$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 \in [0, 3]$.

Proposition 2 is illustrated in Figures 2 and 3, where we simulate taxpayer welfare, evaluated at the optimum, as well as optimal underreporting as a fraction of income, as functions of σ_θ^2, where θ is assumed to follow the same three point distribution that was used previously. The most striking aspect of Figure 2 is how welfare falls much faster for CSEU taxpayers as compared with the EU case, as σ_θ^2 increases. The same pattern emerges for optimal underreporting in Figure 3.

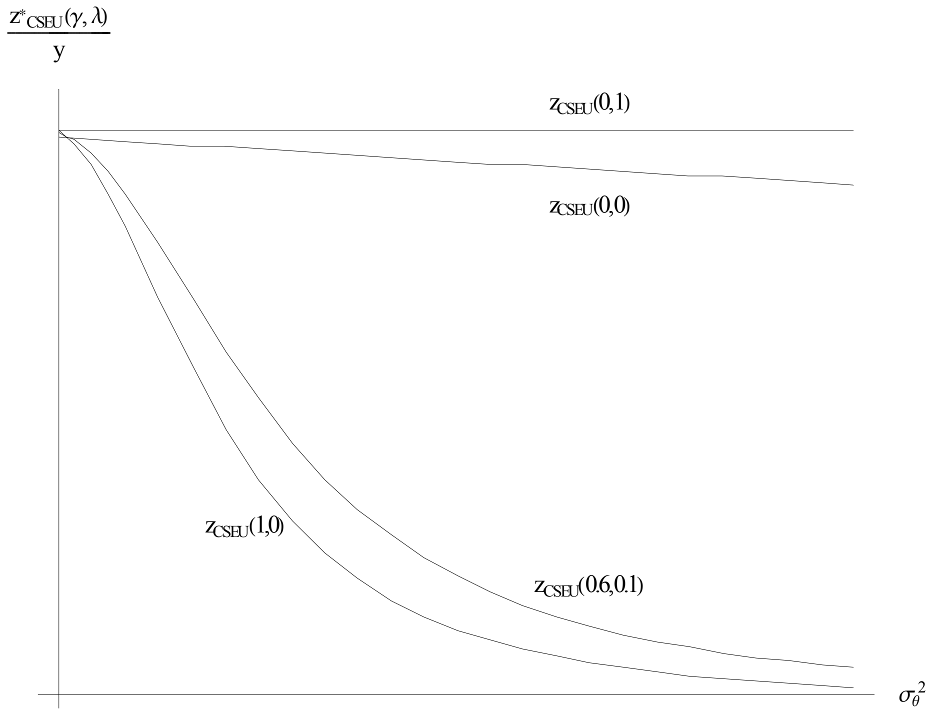

Figure 3: Optimal compliance behaviour under CSEU, as a function of σ_θ^2, for the same parameterization as in Figure 2.

4.1.3 Alternative axiomatics: smooth ambiguity aversion

How robust is the result presented in Proposition 2, in terms of the deterrent effect of ambiguity when the latter takes the form of an MPIR? In order to provide a partial answer to this question, we consider an alternative model that is *not* based on NEO-additive capacities, but which nevertheless formulates an increase in ambiguity in terms of an MPIR. The model in question is that of "smooth ambiguity aversion" recently proposed by Klibanoff, Marinacci, and Mukerji (2004).

In the interest of tractability, we restrict our attention to a situation in which the penalty rate can take on two values, θ_1 and θ_n, with associated probabilities $1-q$ and q. The smooth ambiguity aversion (SAA) evaluation of the tax compliance gamble under consideration is then given by:

$$SAA = \int_0^1 \phi \left(\begin{array}{c} p\left[qu(y-\theta_n tz) + (1-q)u(y-\theta_1 tz)\right] \\ +\left(1-p\right)u(y+tz) \end{array} \right) f\left(q, \rho_q\right) dq, \tag{4}$$

where the probability q is itself a random variable distributed according to the *p.d.f.* $f\left(q, \rho_q\right)$ over the interval $[0,1]$, ρ_q is the parameter of increasing risk of Rothschild and Stiglitz associated with $f\left(q, \rho_q\right)$, and $\phi(.)$ is a twice-differentiable function. Ambiguity-aversion corresponds to $\phi''(.) < 0$. A straightforward application of Lemma 2 allows one to establish the following result:

Proposition 3 *Assume that the taxpayer's preferences can be described by the smooth ambiguity aversion model of Klibanoff, Marinacci, and Mukerji (2004). Then, for an ambiguity-averse taxpayer: (i)* $\frac{dSAA}{d\rho_q} \leqslant 0$*; (ii)* $\frac{dz^*_{SAA}}{d\rho_q} \leqslant 0$.

Proof: See Appendix.

Proposition 3 shows that the results obtained in Proposition 2 concerning the impact of an increase in ambiguity on taxpayer welfare and underreporting are robust to a change in axiomatics, as long as an increase in ambiguity is understood in terms of an MPIR.

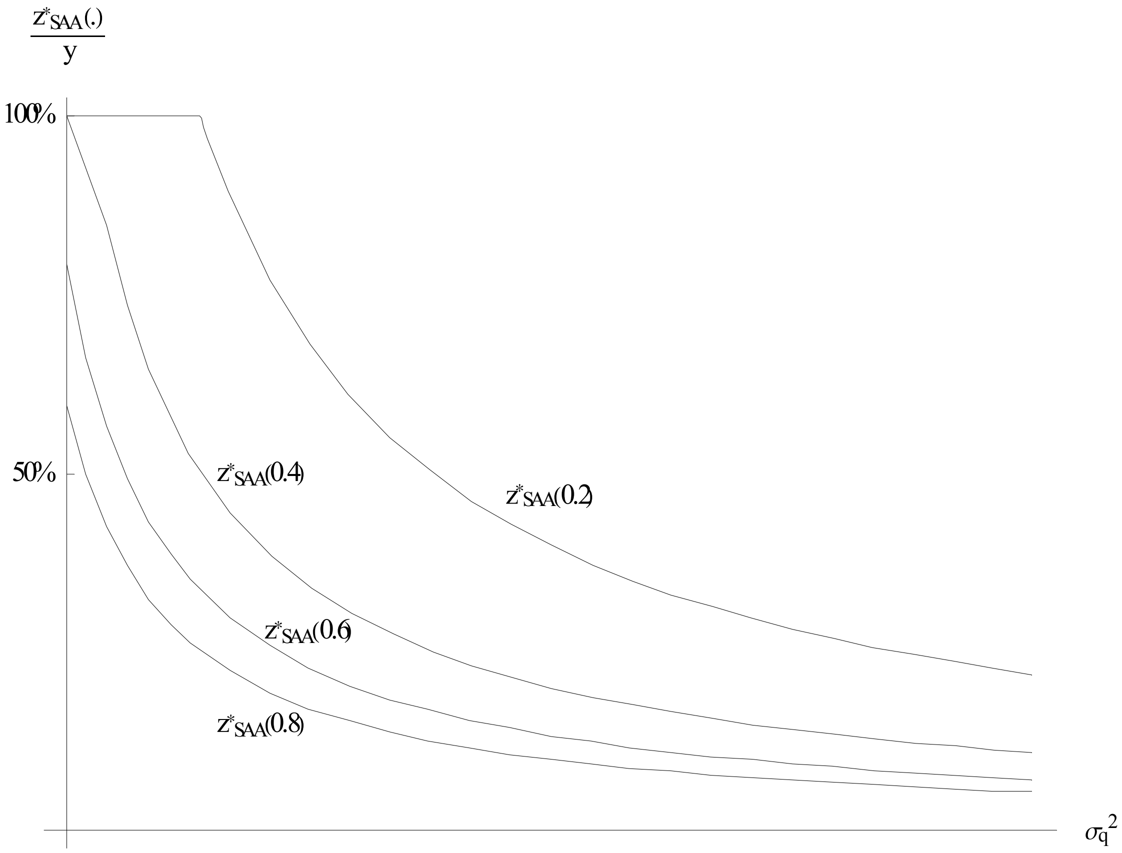

Figure 4: Optimal compliance behaviour as a function of ambiguity, when the probability q of facing penalty rate θ_n is given by the Beta distribution, and: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $\phi(x) = -\frac{e^{-\alpha x}}{\alpha}$, $\alpha = 0.4$, $y = 8$, $t = 0.3$, $p = 0.03$, $\theta_1 = 0$, $\theta_n = 3$, $\mu_q = 0.25$.

In Figure 4, we illustrate Proposition 3 under the assumption that $f(q;.)$ is given by a Beta distribution $\frac{\Gamma(D+B)}{\Gamma(D)\Gamma(B)} q^{D-1}(1-q)^{B-1}, 0 < q < 1, D, B > 0$, with $u(x) = -\frac{e^{-\nu x}}{\nu}$ and $\phi(x) = -\frac{e^{-\alpha x}}{\alpha}$. Explicit computation of the objective function given in (4), optimization with respect to z, and the application of a first-order Taylor expansion to the FOC allows one to solve for optimal underreporting in closed form, which we simulate in Figure 4 for different values of the variance of q ($\sigma_q^2 = \frac{DB}{(D+B)^2(D+B+1)}$) while maintaining $\mu_q = \frac{D}{D+B}$ constant.[11] Each curve corresponds to a different value of the coefficient of absolute ambiguity aversion: the greater the degree of absolute ambiguity aversion, the lower the level of underreporting.

Though the interpretation of an MPIR in the distribution of penalty rates as also corre-

[11] See the Appendix for details of these computations.

sponding to an increase in ambiguity is compelling, it fails to hone in on the most striking aspect of the CSEU with NEO-additive capacities preference functional: the importance of the extrema. We therefore turn to a particular form of MPIR, referred to as a *squeeze*, which will explicitly affect θ_1 and θ_n.

4.2 A *squeeze* in the distribution of θ

Consider an $\alpha-$*squeeze* (Duclos, Esteban, and Ray (2004)) in the distribution of penalty rates. This is a special case of an MPIR, but which *changes* the support of the distribution.

Definition 5 *For $\alpha \in (0, 1]$, an $\alpha-$squeeze of $q(\theta)$ is defined by $q^{\alpha}(\theta) = \frac{1}{\alpha} q\left(\frac{\theta - [1-\alpha]\mu_{\theta}}{\alpha}\right)$.*

The resulting distribution $q^{\alpha}(\theta)$ will be more concentrated around the mean μ_{θ} (which will remain unchanged), its variance will be equal to $\alpha^2\sigma_{\theta}^2$, while the support of the squeezed distribution will be given by $[\alpha\theta_1 + [1-\alpha]\mu_{\theta}, \alpha\theta_n + [1-\alpha]\mu_{\theta}] \subseteq [\theta_1, \theta_n]$.

A squeeze introduces greater heterogeneity in taxpayer response because there can be interesting interactions between the impact of the squeeze on the variance of penalty rates, *versus* its impact on the extrema. For an optimistic taxpayer, for example, an increase in α increases variance, thereby reducing underreporting, while the increase in α also *reduces* the minimal value of the penalty rate, thereby resulting in *greater* underreporting In the limit case of a perfectly optimistic taxpayer ($\gamma = 0, \lambda = 1$), the first effect vanishes altogether, and the impact of an increase in α will be unambiguously to increase underreporting. The converse is true for a perfectly pessimistic taxpayer ($\gamma = 1, \lambda = 0$).

The preceding argument implies that the impact of a change in α is, in the general case, ambiguous, and depends upon the values taken on by γ and λ. One may then state the following:

Proposition 4 *Consider an $\alpha-$squeeze in the distribution of penalty rates. Then: (i) $\frac{d}{d\alpha} z^*_{CSEU}(0,0) \leqslant 0$; (ii) $\frac{d}{d\alpha} z^*_{CSEU}(1,0) \leqslant 0$; (iii) $\frac{d}{d\alpha} z^*_{CSEU}(0,1) \geqslant 0$. Moreover, $\exists\ \gamma^c(\lambda, \alpha)$ such that $\frac{d}{d\alpha} z^*_{CSEU}(\gamma, \lambda) \leqslant 0$, for $\gamma > \gamma^c(\lambda, \alpha)$.*

Proof: See Appendix.

An illustration of Proposition 4 is given in the right-hand panel of Figure 5 where we assume that $q(\theta)$ is given by the arcsin density ($q(\theta) = \frac{1}{\pi\sqrt{\theta(1-\theta)}}, 0 < \theta < 1$), which is represented for different values of α in the left-hand panel. In this case, one can solve explicitly for the threshold value of γ as:

$$\gamma^c(\lambda, \alpha) = \frac{1}{2\alpha^2 p}\left(M - \sqrt{M^2 + 4\alpha^2 p\left(2\left(9p-8\right)\lambda + \alpha\left(1+\lambda\right)\left[\left(3p-2\right)+\alpha p\lambda\right]\right)}\right),$$

where $M = 4(4+\alpha) - p(18 + 6\alpha + \alpha^2)$. As one would expect from Proposition 4, optimal underreporting is increasing in α for a perfectly optimistic CSEU taxpayer, while an EU taxpayer and CSEU taxpayers that display a sufficient degree of pessimism will see their underreporting decrease as α rises.

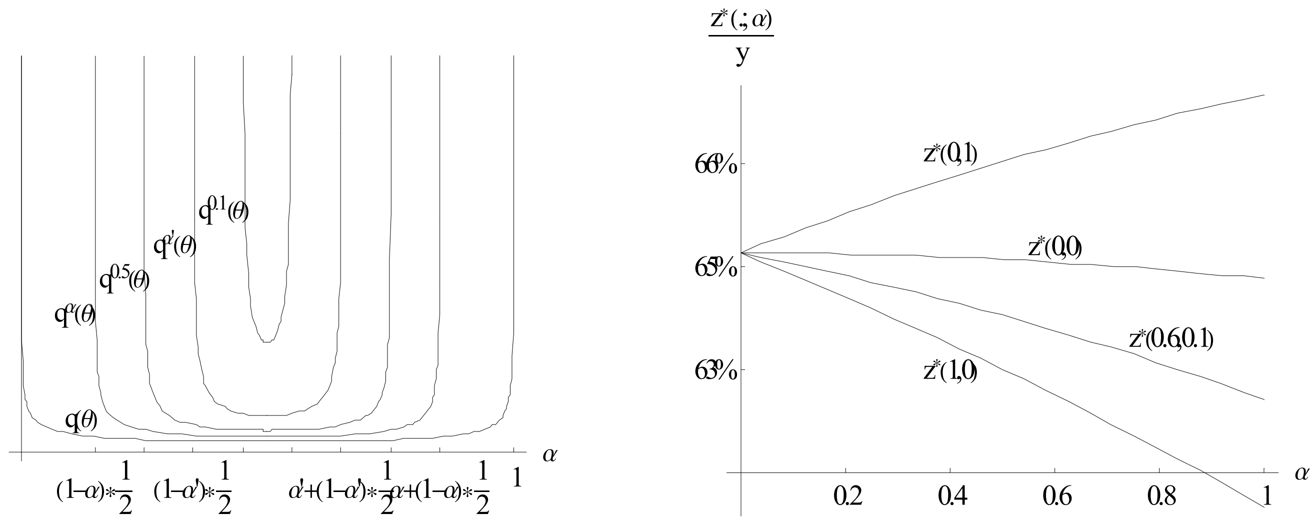

Figure 5: Optimal compliance behaviour for penalty rates distributed according to the arcsin density ($q(\theta) = \frac{1}{\pi\sqrt{\theta(1-\theta)}}, 0 < \theta < 1$), with the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.5$, $y = 10$, $t = 0.3$, $p = 0.03$, $\theta_1 = 0$, $\theta_n = 3$.

4.3 Risk-preserving increases in ambiguity

The characterizations of increases in ambiguity used so far have involved two concepts of MPIRs. We now consider whether it is possible to construct a change in the distribution of penalty rates such that (i) a CSEU decisionmaker will perceive a change in her welfare, whereas (ii) an EU decisionmaker will not. Given that the EU preference functional is affected by changes in the risk of the distribution of penalty rates, we wish to consider changes in the distribution that affect its extrema (θ_1 and θ_n), while leaving the $\sum_{i=1}^{i=n} q_i u\left(y-\theta_i tz\right)$ term unchanged. We shall refer to such a change as a "risk-preserving increase in ambiguity" (RPIA), which we define as follows:

Definition 6 *Consider a random variable distributed according to the p.d.f. $q(\theta,\Delta)$ over the interval $[\theta_1-\Delta,\theta_n+\Delta]$. Then the distribution $q(\theta,\Delta')$ will be said to be more ambiguous than $q(\theta,\Delta)$, for constant risk up to the n^{th} moment about the mean (RPIA(n)), when: (i) $\Delta'>\Delta$, (ii) $\int_{\theta_1-\Delta}^{\theta_n+\Delta}\left(\theta-\mu_\theta\right)^r q(\theta,\Delta)d\theta=\int_{\theta_1-\Delta'}^{\theta_n+\Delta'}\left(\theta-\mu_\theta\right)^r q(\theta,\Delta')d\theta, \forall r=1,...,n.$*

Ensuring that the n^{th} moment about the mean remains constant under an RPIA involves solving an n^{th} degree polynomial equation. For most applications, in which second- or third-order approximations suffice, this procedure is simple to implement. As an illustration, consider once again the three point distribution of penalty rates ($q_1,\theta_1;\ 1-q_1-q_n,\mu_\theta;\ q_n,\theta_n$). Then the distribution given by

$$\left(\begin{array}{c}\frac{q_1q_n(\theta_n-\theta_1)^2}{(\theta_n-\theta_1+2\Delta)[q_n(\theta_n-\theta_1)+(q_1+q_n)\Delta]},\theta_1-\Delta;\\ 1-\frac{q_1q_n(\theta_n-\theta_1)^2}{(\theta_n-\theta_1+2\Delta)[q_n(\theta_n-\theta_1)+(q_1+q_n)\Delta]}-\frac{q_1q_n(\theta_n-\theta_1)^2}{(\theta_n-\theta_1+2\Delta)[q_1(\theta_n-\theta_1)+(q_1+q_n)\Delta]},\mu_\theta;\\ \frac{q_1q_n(\theta_n-\theta_1)^2}{(\theta_n-\theta_1+2\Delta)[q_1(\theta_n-\theta_1)+(q_1+q_n)\Delta]},\theta_n+\Delta\end{array}\right) \tag{5}$$

displays the first two moments which are identical to the initial distribution, but is defined over a wider interval: the minimum value of the penalty rate has fallen by Δ, while the maximum value has increased by Δ. The distribution given in (5) is therefore an RPIA(2) with respect to the initial distribution.

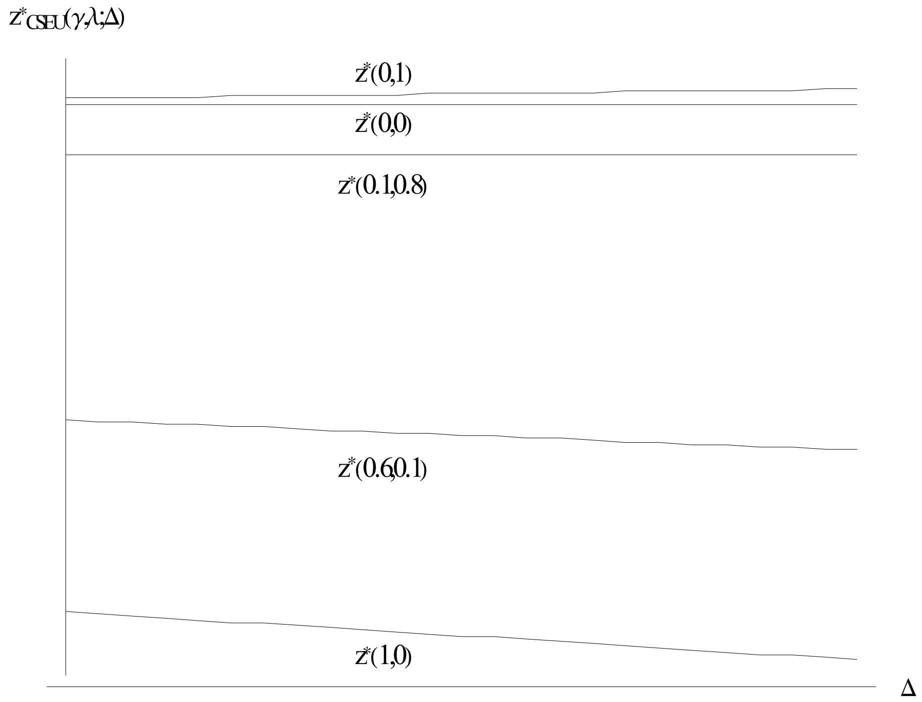

Figure 6: Optimal compliance behaviour, when Δ varies between 0 and 0.2, with the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 10$, $t = 0.3$, $p = 0.03$, $\theta_1 = 0.2$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 = 0.08$.

The effect of an RPIA(2) in the distribution of penalty rates, when the distribution is given by (5), is simulated in Figure 6, as we vary Δ between 0 and 0.2 (the initial value of θ_1 is now equal to 0.2, and we modify the other parameters of the distribution so as to maintain a mean penalty rate of around 25%). As was the case for the $\alpha-$squeeze, there is a threshold configuration of the parameters γ and λ for which the RPIA has no impact on compliance behavior: the increase in underreporting caused by the fall in the minimal penalty rate $(\theta_1 - \Delta)$ as Δ increases, is just offset by the fall in underreporting caused by the increase in the maximal penalty rate $\theta_n + \Delta$. In contrast to the $\alpha-$squeeze, however, there is *no* impact on the variance of the distribution of penalty rates.

In Figure 6, $z^*_{CSEU}(0,1)$ and $z^*_{CSEU}(0.1,0.8)$ display sufficient optimism for underreporting to be increasing in Δ (that $z^*_{CSEU}(0.1,0.8)$ is increasing in Δ is not particularly apparent in visual terms in the Figure). In contrast, $z^*_{CSEU}(0.6,0.1)$ and $z^*_{CSEU}(1,0)$ correspond to taxpayers who are sufficiently pessimistic for underreporting to be decreasing in Δ; $z^*_{CSEU}(0,0) = z^*_{EU}$ is, by the very definition of an RPIA2 in the present context of second-order approximations, invariant to changes in Δ.

The upshot of our consideration of the effect of various measures of ambiguity on compliance behavior is that increasing ambiguity deters underreporting when taxpayers are sufficiently pessimistic, though care must be taken in separating the effect of changes in the risk of the distribution from changes in the extrema. An MPIR, defined in the usual manner, only affects the former, an RPIA only affects the latter, while an $\alpha-$squeeze affects both.

4.4 Gustave Choquet comes to America

How does the CSEU model fare, in terms of its ability to predict true compliance rates over the long term, when compared to the EU benchmark? In Figure 7, we present simulation results for the underreporting rate (z/y) for the US over the past half-century (1947-2002), which we compare to the "true" underreporting rate. The empirical counterpart to z is given here by the US Bureau of Economic Analysis (BEA) "AGI [Adjusted Gross Income] wage gap for wage and salary income", which represents the difference between the BEA's estimate of wages and salaries and taxpayer-reported wages and salaries. This measure is then adjusted to account for "legitimate non-filers" (mainly low-income individuals who are not required to file a tax return) using evidence from the 1988 TCMP study.

The BEA annually estimates a total AGI gap as well as component "gap" measures including employee wages and salaries as well as farm and non-farm proprietor income. Although previous studies have used the total AGI gap as a measure of noncompliance (Crane and Nourzad (1986), Engel and Hines (1999)), the farm and non-farm proprietor gap estimates rely exclusively on tax return data making the total AGI gap a less reliable evasion measure. However, the wage AGI gap is based on independent estimates of wage income reported by employees to the IRS and by employers to state employment agencies. Therefore, due to its high relative degree of accuracy, the wage AGI gap is a preferred measure of income underreporting. Bloomquist (2003) describes the methodology used to derive the modified wage gap.

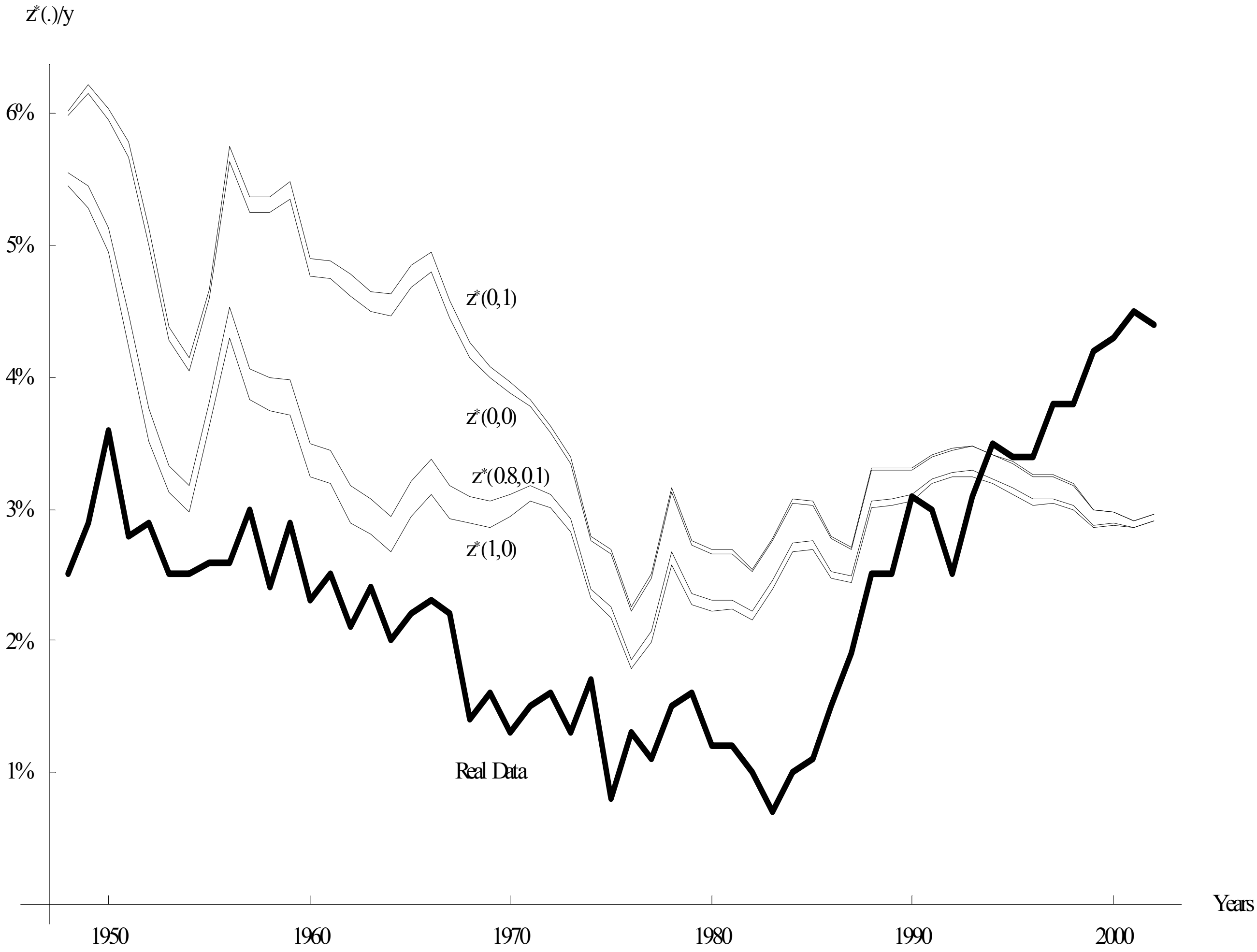

Figure 7: Simulation results on US data, 1947-2000.

The tax rate (t) is a weighted average marginal tax rate on ordinary income (excluding social security and medicare). Income (y) is given by the Census Bureau's Current Population Survey estimate of median wage and salary income. The audit probability p is given by the "face to face" audit rate, as published by the IRS.[12] We shall assume that the penalty rate is distributed according to the three point distribution used earlier. The utility function used in the simulation is of the CARA class.

[12] Sources and adjustments to the tax rate, income and audit variables are provided in Bloomquist (2003).

Correlogram	z real data	z^*_{CSEU} $(0,1)$	z^*_{CSEU} $(0,0)$	z^*_{CSEU} $(0.8,0.2)$	z^*_{CSEU} $(1,0)$
$z^*_{CSEU}(0,1)$	0.227				
$z^*_{CSEU}(0,0)$	0.241	0.999			
$z^*_{CSEU}(0.8,0.2)$	0.411	0.894	0.908		
$z^*_{CSEU}(1,0)$	0.449	0.838	0.856	0.993	
mean	0.023	0.038	0.038	0.031	0.030
std. deviation	0.009	0.010	0.010	0.007	0.007
Correspondence between actual underreporting simulated series					
$z = \underset{(3.20)}{0.015} + \underset{(1.70)}{0.205} \times z^*_{CSEU}(0,1)$			$R^2 = 0.051,\ \sigma = 0.009$		
$z = \underset{(3.02)}{0.014} + \underset{(1.81)}{0.226} \times z^*_{CSEU}(0,0)$			$R^2 = 0.058,\ \sigma = 0.009$		
$z = \underset{(1.24)}{0.006} + \underset{(3.29)}{0.531} \times z^*_{CSEU}(0.8,0.2)$			$R^2 = 0.169,\ \sigma = 0.009$		
$z = \underset{(0.94)}{0.004} + \underset{(3.66)}{0.608} \times z^*_{CSEU}(1,0)$			$R^2 = 0.202,\ \sigma = 0.008$		

Table 1: Actual and simulated underreporting rates for the United States, 1947-2001 (for regression results, t-statistics in parentheses under coefficients)

As should be clear from Figure 7, $z^*_{CSEU}(1,0)$, the most pessimistic specification, performs best in terms of visually tracking the "true" rate of underreporting. This is confirmed formally in Table 1, where we present a simple statistical assessment of the capacity of different parameterizations of the CSEU model to explain the US data. The correlation between $z^*_{CSEU}(1,0)$ and actual data (0.449) is the highest amongst all of the alternatives considered (the corresponding correlation for EU is 0.241) and a regression of the actual value of underreporting on a constant and the simulated series yields an R^2 for $z^*_{CSEU}(1,0)$ of 0.202. The mean simulated rate of underreporting is closest to the real value for $z^*_{CSEU}(1,0)$(the constant in the regression is statistically indistinguishable from zero), though it does tend to underestimate the variance. Broadly speaking, these results show that a pessimistic CSEU model is capable of providing part of the explanation for the tax compliance puzzle in the US case, without the need for the probability distortion functions of the RDEU model that were used in Arcand and Rota-Graziosi (2004).

5 The cost to taxpayers of ambiguity in the tax code

What is the magnitude of the loss incurred by a taxpayer as a result of the ambiguity concerning the penalty rate? In this section, we establish results concerning the premia that taxpayers would be willing to pay for the elimination of risk, ambiguity, or both, in the tax code. Our purpose in doing so is to establish orders of magnitude that will allow us to confront our theoretical predictions with the actual cost of preparing tax returns. Our interpretation of the use of paid preparers in thus motivated by a desire by taxpayers to move from a situation of Knightian uncertainty to one of conventional risk.

An approach which is close in spirit to the one developed here is provided by Fox and Tversky (1995), who explore the "comparative ignorance hypothesis", according to which ambiguity aversion depends upon comparisons with more familiar events or with the opinions of more knowledgeable individuals. We view professional accountants as an extreme example of this hypothesis: such agents do not face uncertainty when they fill in the tax forms of their clients. Moreover, in some countries (such as France), these professionals are able to take out insurances against the risk of errors or lawsuits, thereby protecting their clients.

In the US, a good indicator of the complexity of the tax code is the time required to prepare and file a tax return. Based on IRS estimates, between 1997 and 2003 the time required to complete and file a tax return (both short and long forms) grew approximately 46 percent, from 6.3 hours to 9.1 hours. This increase in time burden results from a combination of new tax law provisions and more taxpayers using Form 1040 (the long form) and fewer taxpayers using the two short forms (Form 1040A and Form 1040EZ). The growth in time burden could explain, in part, the simultaneous increase in the number of taxpayers using paid preparers: in 1997, 51.9 percent of taxpayers used a paid preparer versus 55.6 percent in 2002.

5.1 The risk and ambiguity premia under CSEU with NEO-additive capacities

Before considering the specific case of the tax compliance gamble, it is worthwhile establishing a general result concerning the premia that decisionmakers will be willing to pay in order to eliminate risk and ambiguity, on the one hand, and ambiguity alone, on the other.

Lemma 3 *(Risk and ambiguity premia "in the small" under CSEU with NEO-additive capacities) Consider a gamble* $(q_n, k\varepsilon_n; q_{n-1}, k\varepsilon_{n-1}; ...; q_i, k\varepsilon_i; ...; q_2, k\varepsilon_2; q_1, k\varepsilon_1)$ *with zero expected value* $(\sum_{i=1}^{i=n} q_i\varepsilon_i = 0)$*, variance* $\sigma_\varepsilon^2 = \sum_{i=1}^{i=n} q_i\varepsilon_i^2$*, where* $\varepsilon_n < \varepsilon_{n-1} < ... < 0 < ... < \varepsilon_2 < \varepsilon_1$*, and* k *is a scalar. Then a second order approximation to the risk premium associated with this gamble under CSEU with NEO-additive capacities is given by:*

$$\pi_{CSEU}^{RA}(t;.) = -k(\gamma\varepsilon_n + \lambda\varepsilon_1) + \frac{k^2}{2}\left[\begin{array}{c} \gamma\varepsilon_n^2 + \lambda\varepsilon_1^2 - (\gamma\varepsilon_n + \lambda\varepsilon_1)^2 \\ +(1-\gamma-\lambda)\sigma_\varepsilon^2 \end{array}\right] A(y).$$

Proof: See Appendix.

This Lemma is important in that it shows that it is possible to separate the willingness to pay for the elimination of risk and ambiguity into two components. To see why, note that the standard expression for the risk premium under EU is given by $\pi_{EU}(t;.) = \frac{t^2}{2}\sigma_\varepsilon^2 A(y)$. The difference between the two, $\pi_{CSEU}^{A}(t;.) \equiv \pi_{CSEU}^{RA}(t;.) - \pi_{EU}(t;.)$, represents the willingness to pay for the elimination of ambiguity *alone*:

$$\pi_{CSEU}^{A}(t;.) = -k(\gamma\varepsilon_n + \lambda\varepsilon_1) + \frac{k^2}{2}\left[\gamma\varepsilon_n^2 + \lambda\varepsilon_1^2 - (\gamma\varepsilon_n + \lambda\varepsilon_1)^2 - (\gamma+\lambda)\sigma_\varepsilon^2\right] A(y). \qquad (6)$$

As is true for other non-EU models of decisionmaking, the premia given in Lemma 3 and in equation (6) remain non-zero even in the presence of a linear utility function ($u'' = 0$).[13]

5.2 Premia in the tax compliance gamble

How much would the taxpayer be willing to pay in order to eliminate ambiguity concerning the penalty rates that she faces? Our result in the context of the tax compliance gamble is the following:

[13] For example, see Courtault and Gayant (1998) for the RDEU case.

Proposition 5 *Let* $\Psi = \frac{(1-p-p\mu_\theta-p[\gamma(\theta_n-\mu_\theta)+\lambda(\theta_1-\mu_\theta)])^2}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\left[\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right)+\lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)\right]}$, $\Phi^{RA} = \frac{(1-p-p\mu_\theta)^2}{1-p+p\mu_\theta^2}$ *and* $\Phi^A = \frac{(1-p-p\mu_\theta)^2}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)}$. *Then a second-order approximation to the premium* φ^{RA} *for eliminating risk and ambiguity in the tax compliance gamble is given by:* $\varphi^{RA} = \frac{1}{A(y)}\left(\sqrt{\frac{1-\Psi}{1-\Phi^{RA}}}-1\right)$. *The corresponding premium* φ^A *for the elimination of ambiguity alone is given by:* $\varphi^A = \frac{1}{A(y)}\left(\sqrt{\frac{1-\Psi}{1-\Phi^A}}-1\right)$.

Proof: See Appendix.

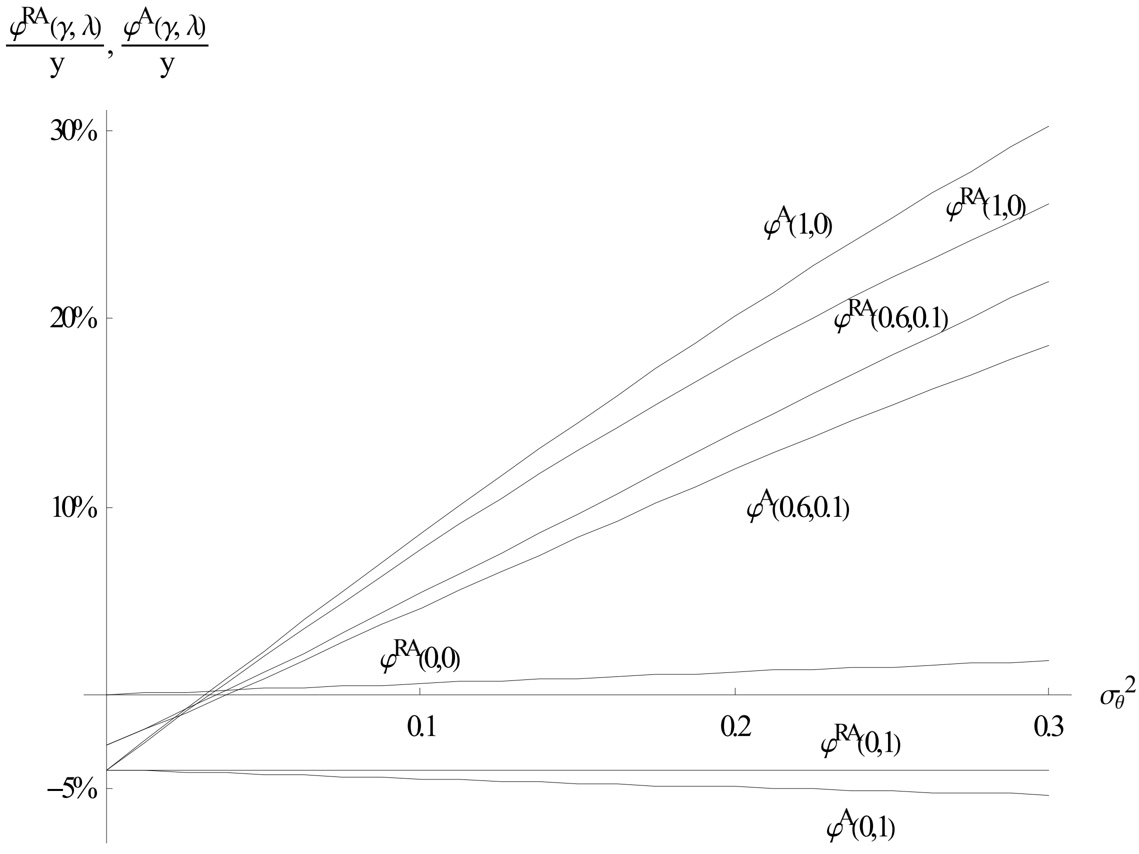

Figure 8: The two risk premia, using the usual three point distribution of penalty rates and the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 1000$, $t = 0.3$, $p = 0.03$, $\theta_1 = 0$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 \in [0, 3]$.

Figure 8 illustrates Proposition 5 by computing the risk and "pure" ambiguity premia for different values of γ and λ, as a function of the variance σ_θ^2 of our usual three-point distribution. The most striking (though unsurprizing, in light of Proposition 5) aspects of the simulation are, first, that the premia are significantly higher for pessimistic CSEU decisionmakers than for the EU case and, second, that the ambiguity portion of the premium appears to account for a major portion of the total. This highlights our interest in the use of professional accountants in preparing tax returns. While one interpretation would view their use as being motivated by a desire to reduce the risk associated with the tax compliance gamble, Figure 8 suggests that, once CSEU preferences are allowed for, risk constitutes a relatively minor concern, compared with ambiguity *per se*.

6 Should tax authorities reduce ambiguity?

Previous theoretical studies, in addition to the present one, have suggested that some tax code complexity might exist by design in order to achieve increased revenues (Scotchmer and Slemrod (1989), Slemrod (1989)). In a controlled experimental study Alm, Jackson, and McKee (1992a) also find that greater uncertainty in either tax due, fine amount or detection probability results in increased compliance. However, when a public good is introduced, reporting compliance actually declined among subject participants. The authors explain that the introduction of a public good causes taxpayer decisions to become interdependent whereas in the absence of a public good compliance decisions are made independently. The authors conclude that by introducing more uncertainty into the tax code "not only are individuals made worse off... but the government may also lose tax revenues."

Much of the theoretical literature emphasizes how complexity elevates the level of uncertainty for the taxpayer but ignores the more practical issue that often increased complexity is accompanied by greater opportunities to evade. For example, Graetz (1999) argues that growing tax law complexity has led to more evasion in the U.S., although he fails to present any evidence of this trend. Highly complicated instructions on tax forms can cause many well-intentioned taxpayers to make mistakes causing the tax authority to expend more money and effort to fix. Greater complexity also gives rise to "gray" areas which might be exploited by some taxpayers and paid preparers. Furthermore, significant and frequent changes to the tax law may lead increasing numbers of taxpayers to rely on paid preparers out of fear they might be paying too much in tax. Therefore, the combination of more evasion opportunities and higher audit costs in addition to the greater burden on taxpayers, both in terms of time and money, tilts the argument against the use of ambiguity for the sole purpose of promoting tax compliance. We formalize this fundamental tradeoff, or at least the deterrence versus increased audit costs portion of it, in the game-theretic context that follows.

6.1 A simple tax compliance game

In this section, we consider a simultaneous move game between the IRS and the taxpayer, which can be thought of empirically as corresponding to a "face-to-face" audit. The IRS chooses the audit rate (p), while the taxpayer chooses her level of underreporting (z). Each player's strategy set is a closed and bounded real-valued interval: $[0,1]$ for the IRS and $[0,y]$ for the taxpayer. Moreover, the IRS is assumed to be risk- and ambiguity-neutral, which corresponds to setting $\gamma = \lambda = 0$ in its CSEU preference functional. In the spirit of Graetz, Reinganum, and Wilde (1986), the IRS's objective function is given by

$$\Pi\left(.;p\right) = t\left(y-z\right) + p\left(1+\mu_\theta\right)tz - c\left(\xi,p\right), \tag{7}$$

where $t\left(y-z\right)$ represents tax receipts on declared income, $p\left(1+\mu_\theta\right)tz$ corresponds to the expected value of penalties collected on non-compliant taxpayers who are audited, while $c\left(\xi,p\right)$ is the cost of implementing an audit rate p, given a level of complexity ξ of the auditing procedure. We assume that $\frac{\partial c(\xi,p)}{\partial \xi} > 0$, $\frac{\partial c(\xi,p)}{\partial p} > 0$, $\frac{\partial^2 c(\xi,p)}{\partial p^2} > 0$ and $\frac{\partial^2 c(\xi,p)}{\partial p \partial \xi} > 0$: the cost of implementing a given audit rate is increasing in the complexity of the auditing procedure and the cost of implementing an audit is increasing and convex in the audit rate one wishes to implement; moreover the marginal impact of complexity on the cost of an audit is increasing in the audit rate one wishes to implement. The convexity assumption ensures that the IRS's objective function is concave in p, while the assumption on the second order cross-partial derivative ensures that the optimal audit rate is a decreasing function of the complexity of the audit, *ceteris paribus*.

We assume that the complexity of auditing is a function of the degree of ambiguity associated with the tax code. Our working hypothesis is that greater ambiguity in the tax code is associated with greater complexity in auditing returns, which we formalize by posing

$$\xi = \xi(\theta_1, \theta_n, \rho) = \int_{\theta_1}^{\theta_n} \zeta(\theta) q(\theta, \rho) d\theta,$$

where $\frac{d^2\zeta(\theta)}{d\theta^2} > 0$ and ρ is the parameter of increasing risk of Rothschild and Stiglitz. By a trivial application of Lemma 2, this specification implies that an increase in the ambiguity of the tax code modelled as an MPIR of $q(\theta, \rho)$ will increase the complexity of an audit, while an $\alpha-$squeeze of $q(\theta)$ as given in Definition 5 will decrease it. The key issue in determining the impact on the equilibrium level of underreporting in the game that follows will then be the magnitude of $\frac{d\xi(\theta_1,\theta_n,\rho)}{d\rho} = \int_{\theta_1}^{\theta_n} \frac{d^2\zeta(\theta)}{d\theta^2} \left(\int_{\theta_1}^{\theta} Q_\rho(x, \rho) dx \right) d\theta > 0$. Let

$$p^* \left(.; z\right) = \underset{p \in [0,1]}{\arg\max} \Pi \left(.; p\right) \tag{8}$$

denote the solution to the IRS's maximization program. By implicit differentiation of the FOC that corresponds to (8), it is then immediate that $\frac{\partial p^*(.;z)}{\partial z} > 0$. The taxpayer's optimal compliance behavior is characterized in Proposition 1, from which it is straightforward to show that $\frac{\partial z^*_{CSEU}(.)}{\partial p} < 0$. The action space of each player is continuous, and the payoff functions $\Pi\left(.; p\right)$ and $CSEU\left(\gamma, \lambda; z\right)$ are continuous.

The concept of a Nash equilibrium has been extended to CSEU axiomatics by Dow and Werlang (1994), Lo (1996), Marinacci (2000), and Eichberger and Kelsey (2000). We use the concept of Equilibrium Under Ambiguity (EUA) developed by Eichberger, Kelsey, and Schipper (2004) to establish the existence of an EUA.[14] The EUA values of underreporting and the penalty rate, denoted by (p^{EUA}, z^{EUA}), are then given by the solution in (p, z) of the

[14] See their Proposition 3.2, p. 11.

pair of equations given by (8) and optimal compliance behavior as characterized in Proposition 1. Figure 9 illustrates the IRS's reaction function, as well as that of the taxpayer, for different values of γ and λ, for a parameterization that is slightly different from that used previously.[15] As would be expected, greater pessimism on the part of taxpayers leads to lower levels of z^{EUA}, as well as to lower levels of p^{EUA}.

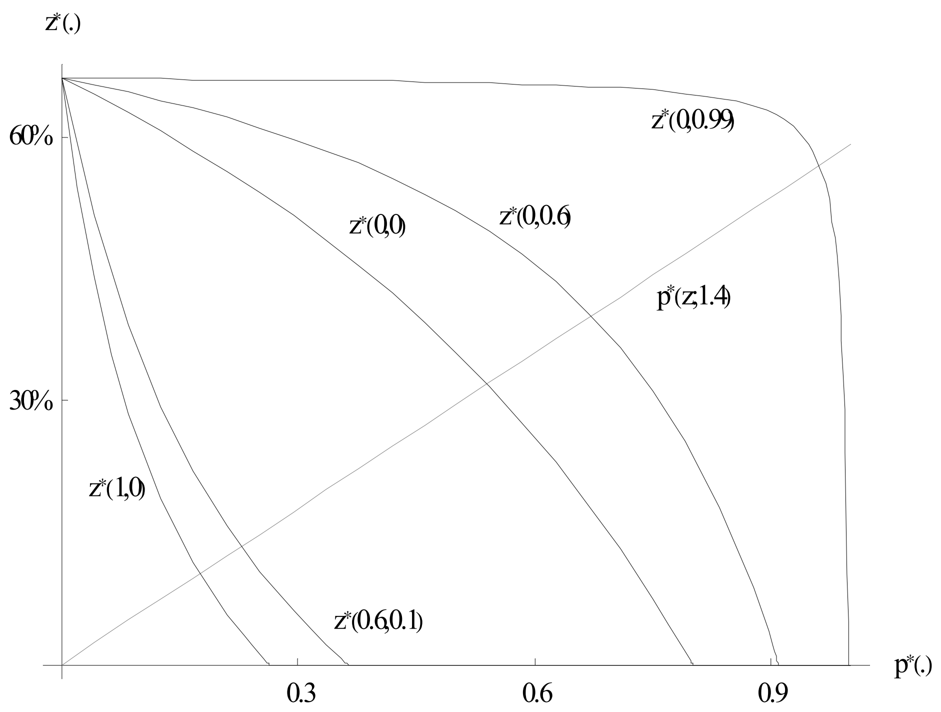

Figure 9: Reaction functions under the parameterization: $u(x) = -\frac{e^{-Rx}}{R}$, $R = 0.05$, $\mu_\theta = 1.2$, $\theta_n = 2$, and $\sigma_\theta = 0.025$.

6.2 Changes in the distribution of penalty rates and equilibrium underreporting

In this section, we consider the impact of changes in the distribution of penalty rates on the equilibrium level of underreporting. As in section 4, we consider three different changes in the distribution of θ: an MPIR, an $\alpha-$squeeze, and an RPIA. We conclude by provid-

[15] This change in parametrization is made purely for esthetic reasons.

ing an initial assessment of the ability of our theoretical construct to mimic the empirical relationship linking tax code complexity to underreporting and the willingness to pay for accountants' services.

6.2.1 An MPIR, an $\alpha-$squeeze and an RPIA

Let $\widetilde{\omega}(\theta,\rho)$ be the value of $\omega(\theta) \equiv \frac{d^2\zeta(\theta)}{d\theta^2}$ such that

$$\frac{d\xi(\theta_1,\theta_n,\rho)}{d\rho} = \int_{\theta_1}^{\theta_n} \widetilde{\omega}(\theta,\rho)\left(\int_{\theta_1}^{\theta} Q_\rho(x,\rho)dx\right)d\theta = \frac{\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial\rho}\frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p^2}}{\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p}\frac{\partial^2 c(\xi,p^{EUA})}{\partial p\partial\xi}}. \tag{9}$$

While apparently complex, the interpretation of equation (9) is extremely straightforward: the function $\omega(\theta)$ characterizes the degree of convexity in θ of $\zeta(\theta)$ and therefore the sensitivity of the complexity ξ of an audit with respect to an MPIR. The net impact on z^{EUA} of an MPIR is the result of two effects of opposite sign. On the one hand, an MPIR reduces underreporting, by Proposition 2(ii). On the other, an MPIR decreases the IRS's optimal audit rate (because ξ increases), thereby leading to higher underreporting. The outcome in terms of $\frac{dz^{EUA}}{d\rho}$ then depends upon the relative magnitudes of these two effects. The value of $\omega(\theta)$ implicitly defined in (9) corresponds to a situation in which the two effects exactly cancel out and in which an MPIR has *no* effect on z^{EUA}. The following Proposition formalizes this intuition by characterizing the impact of an increase in ambiguity on z^{EUA}, as modelled by an MPIR in the distribution of penalty rates:

Proposition 6 *Consider the EUA* (p^{EUA}, z^{EUA}) *of the preceding simultaneous move game. Then sign* $\left[\frac{dz^{EUA}}{d\rho}\right] = sign\left[\omega(\theta) - \widetilde{\omega}(\theta,\rho)\right]$.

Proof: See Appendix.

Proposition 6 is illustrated in Figure 10 (in the Appendix), where we use our usual three point distribution and specify $\zeta(\theta) = \theta^\eta, \eta > 1$ and $c(\xi,p) = \frac{c}{2}\xi^2p^2$. We plot $\frac{z^{EUA}}{y}$ as a function of σ^2_θ, for η ranging from 1 to 1.3. In the case of $\eta = 1$, the MPIR has *no* effect on the cost of the audit (because $\frac{d^2\zeta(\theta)}{d\theta^2} = 0$, by Lemma 2(i)) and the equilibrium outcome stems from the "pure" effect of the change in σ_θ^2 on underreporting: the curves in the four panels of Figure 10 are therefore all downward-sloping for $\eta = 1$ (as was the case in Figure 3). As the degree of convexity of $\zeta(\theta)$ increases, the shape of the curve changes. For the EU case or a relatively optimistic taxpayer ($(\gamma, \lambda) = (0, 0.9)$), $\frac{z^{EUA}}{y}$ is increasing in σ_θ^2 (see the two left-hand panels in Figure 10). In contrast, the two right-hand panels of Figure 10 assume that the taxpayer is relatively pessimistic. For $(\gamma, \lambda) = (0.6, 0.1)$, for example, a value of $\eta = 1.3$ leads to $\frac{z^{EUA}}{y}$ being initially increasing in σ_θ^2 (for very low levels of σ_θ^2), with the threshold level being rapidly attained: the curves become downward-sloping thereafter.

In Figure 11 (in the Appendix) we carry out the same exercise, but for an $\alpha-$squeeze with the distribution of penalty rates given by the arcsin density. We consider values of η that range from 1 to 5. Because of the effect of the $\alpha-$squeeze on the extrema of the distribution of penalty rates, which affects not only underreporting (as seen earlier) but the cost of an audit as well, the interactions are more complex, though the pattern that emerges graphically is always the same: regardless of the parameterization in terms of γ and λ, $\frac{z^{EUA}}{y}$ is *always* an inverted U-shaped function of α. The contrast with the partial equilibrium impact of an increase in α on underreporting given in Figure 5 is striking. Consider the case of a pessimistic consumer (top right-hand panel in Figure 11), with $(\gamma, \lambda) = (1, 0)$: in this case, Figure 5 showed that the partial equilibrium impact of an increase in α was to *decrease* underreporting in an unambiguous manner. In the game-theoretic context, on the other hand, this pattern only emerges once a threshold level of α has been crossed: for low levels of α, the decrease in the cost of an audit caused by the $\alpha-$squeeze (and thus the increase in the underlying audit probability) is sufficient to more than offset the tendency of the taxpayer to underreport less, the equilibrium result being that $\frac{z^{EUA}}{y}$ is increasing in α for low values of the latter.

Finally, in Figure 12 (in the Appendix) and 13 (below), we consider the impact of an RPIA(2), as given in (5), on $\frac{z^{EUA}}{y}$, for our three point distribution of penalty rates. In Figure 12, we plot $\frac{z^{EUA}}{y}$ as a function of Δ for $\eta = 1, 2, 3$. Note, for $\eta = 1$ or $\eta = 2$, that the definition of an RPIA implies that there is *no* impact of a change in Δ on the cost of an audit. The impact of an increase in Δ on the equilibrium outcome is therefore entirely determined by the partial equilibrium effect on underreporting that was considered in Figure 6. This implies that, for the optimistic taxpayers considered in two left-hand panels of Figure 12, $\frac{z^{EUA}}{y}$ is increasing in Δ, whereas the opposite holds for the pessimistic taxpayers considered in the two right-hand panels.[16] This is highlighted in Figure 13, where we restrict our attention to $\eta = 2$ and where the impact of a rise in Δ follows exactly the same pattern as in Figure 6.

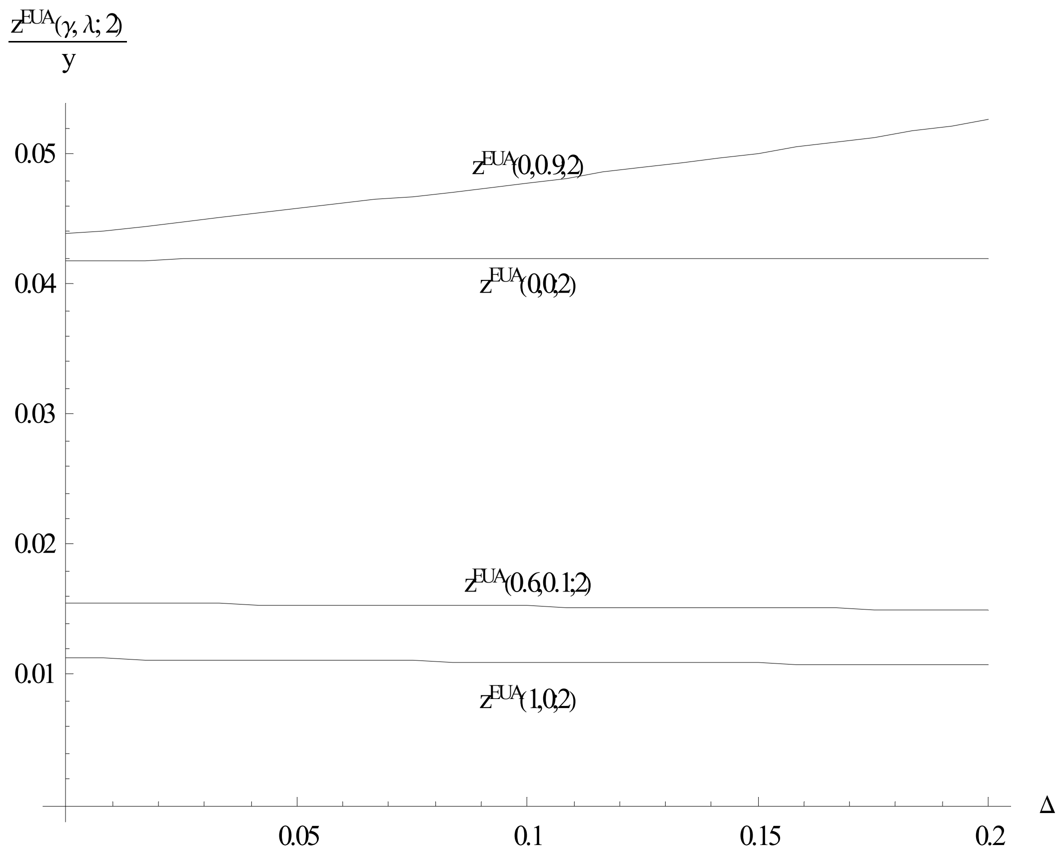

Figure 13: $\frac{z^{EUA}(.)}{y}$, as Δ varies between 0 and 0.2, with the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 10$, $t = 0.3$, $p = 0.03$, $\theta_1 = 0.2$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 = 0.08$.

[16] Note that while an RPIA(2) *does* affect the cost of an audit when $\eta = 3$, its impact is not sufficiently great to overturn the pattern that emerges in the two preceding cases, and the effect of an increase in Δ remains consonant with that uncovered in a partial equilibrium setting.

Adjusted gross income category (AGI)	Mean AGI	Complexity indicator		Predicted tax change (IRS estimate of evasion)	Willingness to pay % of AGI	Estimated cost of audit
		mean	median			
	y	$\rho, \sigma_\theta^2, \rho_q, \frac{1}{\alpha}, \Delta, \xi$		z	φ^{RA} or φ^{A}	$c(\xi, p)$
\$0 to < \$15K	\$ 7 412	1,58	1	\$ 167	0,99%	\$474
\$15K to < \$30K	\$ 21 965	1,68	1	\$ 286	0,50%	\$495
\$30K to < \$45K	\$ 36 927	1,96	2	\$ 370	0,36%	\$559
\$45K to < \$60K	\$ 52 075	2,23	2	\$ 466	0,32%	\$628
\$60K to < \$90K	\$ 73 036	2,45	2	\$ 540	0,29%	\$691
\$90K to < \$120K	\$102 845	2,58	2	\$ 909	0,26%	\$731
\$120K or more	\$334 194	2,82	3	\$2 221	0,13%	\$811
total	\$ 50 477	1,94	1	\$ 451	0,29%	\$554

Table 2: Taxable income, complexity, the willingness to pay for accountants' services, and the cost of an audit: the US in the year 2000

6.3 Empirical evidence (preliminary and incomplete)

Though historical time series evidence does not exist in terms of the cost to taxpayers of preparing their returns, or of their use of professional accountants, some data do exist on a cross-sectional basis, for different levels of taxable income. Data also exist that allow one to link complexity to the level of underreporting. The IRS's Compliance Information Research System (CRIS) provides an indicator of the complexity of tax returns. In turn, Guyton, O'Hare, Stavrianos, and Toder (2003) construct a measure of the "out of pocket money burden" of preparing a tax return, which allows one to establish a relationship between level of income, the complexity of preparing a return, and a rough measure of the willingness to pay for accountants' services. A coherent measure of z in this context is provided by the "predicted tax change", an in-house IRS measure of evasion (this is not the same as the AGI wage gap measures presented earlier). The available data are presented in Table 2.

6.3.1 Complexity and the cost of an audit

With respect to the relationship between tax code complexity and the cost of an audit, evidence is available that allows one to assess the validity of the assumptions posed earlier. The time to complete the average audit of an individual non-business taxpayer grew from

9 hours in Fiscal Year (FY) 1997 to 19 hours in FY2003, a jump of over 100 percent. Much of this increase can be attributed to passage of the 1998 taxpayer bill of rights that required the IRS to allow taxpayers more time to settle amounts due. However, some of this increase can be attributed to added complexity of the returns (GAO 2001). How much of this increase is due to increased complexity would be difficult to identify and no studies have been conducted to separate the various contributing factors. However, if we assume that of the average increase of 10 hours audit time 2 hours is due to increased complexity and further assume an average examiner cost of \$50 per hour (including benefits, leave, etc.), then the added complexity would boost per return audit cost by \$100. Based on the data presented in Table 2, the relationship between complexity and the cost of an audit for FY2000 is given by

$$c = \$50 \times 4.7829e^{0.4331\xi}, R^2 = 0.8312.$$

This relationship is convex, as assumed in our theoretical discussion.

6.3.2 Complexity and the willingness to pay for accountant's services

The estimated relationship between the cost of preparing a return (and thus the willingness to pay for accountant services) and complexity is given by

$$\varphi^{RA} = \$11.4e^{1.2377\xi}, R^2 = 0.9538.$$

Though this increasing relationship between complexity and an empirical proxy for φ^{RA} mimics the theoretical relationship plotted in Figure 8, the latter was a partial equilibrium result that did not include the interaction with the IRS in the context of the tax-compliance game. Further theoretical results are needed in order to establish whether such a pattern would emerge as the equilibrium outcome.

6.3.3 Complexity and underreporting (incomplete)

INCOMPLETE: contrast relationship using mean versus relationship using median AGI

7 Concluding remarks

This paper has shown that ambiguity, in the sense of Knightian uncertainty, concerning the penalty rate, generates a tax compliance problem that is very different from the EU or RDEU cases. In the context of a CSEU model with the NEO-additive capacities of Chateauneuf, Eichberger, and Grant (2004), we have characterized optimal compliance behavior and studied how the latter varies as a function of various measures of ambiguity.

We have shown that taxpayers may suffer significant welfare losses as a result of the ambiguity concerning penalty rates, and would be willing to pay a substantial fraction of their income in order to eliminate it. Most of this willingness to pay appears to stem from pure ambiguity concerns, *not* from risk.

Finally, in the context of a simple tax compliance game where the IRS sets the audit probability and taxpayers choose their level of compliance, we have shown that the key parameter, in terms of the effect on underreporting of a simpler or a more complex tax code, is the increase in the cost of an audit associated with greater complexity. When the increase in the cost of an audit associated with increasing complexity is sufficiently high, equilibrium underreporting will be an increasing function of complexity. In this case, it would clearly be in the interest of the IRS to simplify the tax code.

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A Appendix

A.1 Proof of Proposition 1: optimal compliance behavior

The solution to the taxpayer's optimization problem is given by

$$z^*_{CSEU} \equiv \underset{\{z \geqslant 0\}}{\arg\max} CSEU\left(\gamma, \lambda; z\right),$$

where $CSEU\left(\gamma, \lambda\right)$ is defined in (1). The necessary First Order Condition (FOC) that implicitly characterizes optimal compliance behavior is given by:[17]

$$p\left[\begin{array}{c} -\theta_n t\gamma u'\left(y - \theta_n t z^*_{CSEU}\right) - \theta_1 t\lambda u'\left(y - \theta_1 t z^*_{CSEU}\right) \\ -\left(1-\gamma-\lambda\right) t \sum_{i=1}^{n} \theta_i q_i u'\left(y - \theta_i t z^*_{CSEU}\right) \end{array}\right] + \left(1-p\right) t u'\left(y + t z^*_{CSEU}\right) = 0.$$

Substituting the first-order Taylor expansions $u'(y-\theta_i t z^*_{CSEU}) \approx u'(y) - \theta_i t z^*_{CSEU} u''(y)$ and $u'(y+tz^*_{CSEU}) \approx u'(y) + t z^*_{CSEU} u''(y)$ into the FOC allows one to write:

$$\begin{aligned} 0 \quad = \quad & \left[1-p-p\left(\theta_n\gamma + \theta_1\lambda + \left(1-\gamma-\lambda\right)\mu_\theta\right)\right] u'\left(y\right) \\ & + \left[1-p+p\left(\theta_n^2\gamma + \theta_1^2\lambda + \left(1-\gamma-\lambda\right)\left(\mu_\theta^2 + \sigma_\theta^2\right)\right)\right] t z^*_{CSEU} u''\left(y\right), \end{aligned} \tag{A.10}$$

where $\mu_\theta = \sum_{i=1}^{i=n} q_i\theta_i$, $\sigma^2_\theta = \sum_{i=1}^{i=n} q_i\theta_i^2 - \mu_\theta^2$. An interior solution will exist if and only if:

$$\left[1-p-p\left(\theta_n\gamma + \theta_1\lambda + \left(1-\gamma-\lambda\right)\mu_\theta\right)\right]\left[1-p+p\left(\theta_n^2\gamma + \theta_1^2\lambda + \left(1-\gamma-\lambda\right)\left(\mu_\theta^2 + \sigma_\theta^2\right)\right)\right] > 0,$$

which is equivalent to the condition that:

$$p < p_{CSEU}\left(\gamma, \lambda\right) = \frac{1}{1 + \theta_n\gamma + \theta_1\lambda + \left(1-\gamma-\lambda\right)\mu_\theta}. \tag{11}$$

[17] The SOC is satisfied because of the concavity of $u\left(.\right)$.

Combining (10) and (11) implies that:

$$z^*_{CSEU}(\gamma,\lambda) = \begin{cases} \frac{1}{tA(y)}\left(\frac{1-p-p(\theta_n\gamma+\theta_1\lambda+(1-\gamma-\lambda)\mu_\theta)}{1-p+p\left(\theta_n^2\gamma+\theta_1^2\lambda+(1-\gamma-\lambda)\left(\mu_\theta^2+\sigma_\theta^2\right)\right)}\right), \text{ if } p < p_{CSEU}(\gamma,\lambda) \\ 0, \text{ otherwise} \end{cases} \blacksquare \tag{12}$$

A.2 Proof of Proposition 2: impact of an MPIR on welfare and optimal compliance behavior

Straightforward differentiation yields:

$$\frac{\partial^2}{\partial\theta^2}u(y-\theta tz) = t^2z^2u''(y-\theta tz) < 0,$$

and

$$\frac{\partial^3}{\partial\theta^2\partial z}u(y-\theta tz) = t^2z\left[2u''(y-\theta tz) - \theta tzu'''(y-\theta tz)\right] < 0.$$

Application of Lemma 2 then yields the result given in the Proposition. Note also that a second-order Taylor expansion allows us to write the objective function of the taxpayer as:

$$\begin{aligned} &CSEU(\gamma,\lambda;z) = u(y) + u'(y)t \\ &\times\left(\begin{array}{c} \left[1-p-p\mu_\theta - p\left[\gamma\left(\theta_n-\mu_\theta\right)+\lambda\left(\theta_1-\mu_\theta\right)\right]\right]z^*_{CSEU} \\ -\frac{1}{2}\left[1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\left[\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right)+\lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)\right]\right]z^{*2}_{CSEU}tA(y)\end{array}\right).\end{aligned}$$

Evaluating this expression at the optimal value of z^*_{CSEU} (as given in Proposition 1) then yields:

$$\begin{aligned} &CSEU(\gamma,\lambda;z) \\ =\ & u(y) + \frac{u'(y)}{2A(y)}\frac{\left(1-p-p\mu_\theta-p\left[\gamma\left(\theta_n-\mu_\theta\right)+\lambda\left(\theta_1-\mu_\theta\right)\right]\right)^2}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\left[\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right)+\lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)\right]}. \end{aligned} \tag{A.13}$$

This expression will be decreasing in σ_θ^2 as long as $1-\gamma-\lambda>0$, as will z^*_{CSEU} as given in Proposition 1.$\blacksquare$

A.3 Proof of Proposition 3: impact of an MPIR under smooth ambiguity aversion

The FOC which implicitly defines the optimal level z^*_{SAA} of underreporting is given by:

$$\begin{aligned} 0 \ =\ & t\left[\begin{array}{c}(1-p)\,u'(y+tz^*_{SAA}) \\ -p\left[\theta_nqu'(y-\theta_ntz^*_{SAA})+\theta_1t(1-q)u'(y-\theta_1tz^*_{SAA})\right]\end{array}\right] \\ &\times\int_0^1\phi'\left(\begin{array}{c}p\left[qu(y-\theta_ntz^*_{SAA})+(1-q)u(y-\theta_1tz^*_{SAA})\right] \\ +(1-p)\,u(y+tz^*_{SAA})\end{array}\right)f\left(q,\rho_q\right)dq, \end{aligned}$$

which implies that:

$$(1-p)\,u'(y+tz^*_{SAA}) - p\left[\begin{array}{c}\theta_nqu'(y-\theta_ntz^*_{SAA}) \\ +\theta_1t(1-q)u'(y-\theta_1tz^*_{SAA})\end{array}\right] = 0.$$

Straightforward differentiation of the integrand of the objective function twice with respect to q yields:

$$\begin{aligned}
&\frac{\partial^2}{\partial q^2}\phi\left(p\left[\begin{array}{c} qu(y-\theta_n tz^*_{SAA}) \\ +(1-q)u(y-\theta_1 tz^*_{SAA}) \end{array}\right]+(1-p)\,u(y+tz^*_{SAA})\right) \\
=\ & p^2\left[\begin{array}{c} u(y-\theta_n tz^*_{SAA}) \\ -u(y-\theta_1 tz^*_{SAA}) \end{array}\right]^2 \phi''\left(\begin{array}{c} p\left[\begin{array}{c} qu(y-\theta_n tz^*_{SAA}) \\ +(1-q)u(y-\theta_1 tz^*_{SAA}) \end{array}\right] \\ +(1-p)\,u(y+tz^*_{SAA}) \end{array}\right),
\end{aligned}$$

which, applying Lemma 2(i), implies that $\frac{dSAA}{d\rho_q}<0$, as long as the taxpayer is ambiguity-averse ($\phi''<0$). Differentiating the preceding expression with respect to z yields:

$$\begin{aligned}
&\frac{\partial^3}{\partial z\partial q^2}\phi\left(p\left[\begin{array}{c} qu(y-\theta_n tz^*_{SAA}) \\ +(1-q)u(y-\theta_1 tz^*_{SAA}) \end{array}\right]+(1-p)\,u(y+tz^*_{SAA})\right) \\
=\ & tp^2\left[2\left[\begin{array}{c} \theta_1 u'(y-\theta_1 tz^*_{SAA}) \\ -\theta_n u'(y-\theta_n tz^*_{SAA}) \end{array}\right]\left[\begin{array}{c} u(y-\theta_n tz^*_{SAA}) \\ -u(y-\theta_1 tz^*_{SAA}) \end{array}\right]\phi''(.)\right] \\
&+tp^2\left[\left(p\left[\begin{array}{c} -\theta_n qu'(y-\theta_n tz^*_{SAA}) \\ -\theta_1(1-q)u'(y-\theta_1 tz^*_{SAA}) \end{array}\right]+(1-p)\,u'(y+tz^*_{SAA})\right)\phi'''(.)\right].
\end{aligned}$$

Substituting from the FOC yields:

$$\begin{aligned}
&\frac{\partial^3}{\partial z\partial q^2}\phi\left(p\left[\begin{array}{c} qu(y-\theta_n tz^*_{SAA}) \\ +(1-q)u(y-\theta_1 tz^*_{SAA}) \end{array}\right]+(1-p)\,u(y+tz^*_{SAA})\right) \\
=\ & 2tp^2\left[\begin{array}{c} \theta_1 u'(y-\theta_1 tz^*_{SAA}) \\ -\theta_n u'(y-\theta_n tz^*_{SAA}) \end{array}\right]\left[\begin{array}{c} u(y-\theta_n tz^*_{SAA}) \\ -u(y-\theta_1 tz^*_{SAA}) \end{array}\right]\phi''(.).
\end{aligned}$$

A first-order Taylor expansion of the first element on the RHS of this expression allows one to rewrite it as:

$$\begin{aligned}
&\frac{\partial^3}{\partial z\partial q^2}\phi\left(p\left[\begin{array}{c} qu(y-\theta_n tz^*_{SAA}) \\ +(1-q)u(y-\theta_1 tz^*_{SAA}) \end{array}\right]+(1-p)\,u(y+tz^*_{SAA})\right) \\
=\ & 2tp^2 2\left[u'(y)\right]^2(\theta_n-\theta_1)^2\left[1+(\theta_n+\theta_1)A(y)tz\right]tz^*_{SAA}\phi''(.)<0.
\end{aligned}$$

By Lemma 2(ii), it follows that $\frac{dz^*_{SAA}}{d\rho_q}<0$.■

A.4 An explicit second-order approximation to optimal underreporting under smooth ambiguity aversion, with a Beta distribution and negative-exponential utility and ambiguity functionals

$$z^*_{SAA}=\frac{e^{RY}\left(DH(\theta_1)+BH(\theta_n)\right)}{t\left(\frac{D(1+D)H(\theta_1)^2+B(1+B)H(\theta_n)^2+2BDH(\theta_1)H(\theta_n)}{1+B+D}+e^{RY}R\left[\begin{array}{c} D\left(1+p\left(\theta_1^2-1\right)\right) \\ +B\left(1+p\left(\theta_n^2-1\right)\right) \end{array}\right]\right)},$$

where $H(\theta_i)=1-p-p\theta_i$, $\forall i=1,n$.

A.5 Proof of Proposition 4: impact of a squeeze on welfare and optimal compliance behavior

Consider the expression for the welfare of the taxpayer at the optimum, given in (13). Applying the α-squeeze to this expression yields:

$$\begin{aligned} & CSEU\left(\gamma,\lambda;z\right) \\ = \quad & u(y)+\frac{u'(y)}{2A(y)}\frac{\left(1-p-p\mu_{\theta}-p\left[\begin{array}{c}\gamma\left(\left(\alpha\theta_{n}+\left[1-\alpha\right]\mu_{\theta}\right)-\mu_{\theta}\right)\\ +\lambda\left(\left(\alpha\theta_{1}+\left[1-\alpha\right]\mu_{\theta}\right)-\mu_{\theta}\right)\end{array}\right]\right)^{2}}{1-p+p\left(\mu_{\theta}^{2}+\alpha^{2}\sigma_{\theta}^{2}\right)+p\left[\begin{array}{c}\gamma\left(\left(\alpha\theta_{n}+\left[1-\alpha\right]\mu_{\theta}\right)^{2}-\mu_{\theta}^{2}-\alpha^{2}\sigma_{\theta}^{2}\right)\\ +\lambda\left(\left(\alpha\theta_{1}+\left[1-\alpha\right]\mu_{\theta}\right)^{2}-\mu_{\theta}^{2}-\alpha^{2}\sigma_{\theta}^{2}\right)\end{array}\right]}.\end{aligned}$$

Applying the same principle to the second-order approximation given in Proposition 1 yields:

$$z_{CSEU}^{*}\left(\gamma,\lambda\right)=\frac{1}{tA(y)}\frac{1-p-p\left(\begin{array}{c}\gamma\left(\alpha\theta_{n}+\left[1-\alpha\right]\mu_{\theta}\right)\\ +\lambda\left(\alpha\theta_{1}+\left[1-\alpha\right]\mu_{\theta}\right)+\left(1-\gamma-\lambda\right)\mu_{\theta}\end{array}\right)}{1-p+p\left(\begin{array}{c}\gamma\left(\alpha\theta_{n}+\left[1-\alpha\right]\mu_{\theta}\right)^{2}\\ +\lambda\left(\alpha\theta_{1}+\left[1-\alpha\right]\mu_{\theta}\right)^{2}+\left(1-\gamma-\lambda\right)\left(\mu_{\theta}^{2}+\alpha^{2}\sigma_{\theta}^{2}\right)\end{array}\right)}.$$

Straightforward but tedious differentiation of these expressions with respect to α establishes the Proposition.■

A.6 An α-squeeze when penalty rates are distributed according to the arcsin distribution

$$z_{CSEU}^{*}\left(\gamma,\lambda\right)=\frac{1}{tA(y)}\frac{4\left(2-p\left[3+\alpha\left(\gamma-\lambda\right)\right]\right)}{8+p\left(-6+\alpha\left(\alpha+4\gamma\left(1+\lambda\right)-\left(4-\alpha\right)\lambda\right)\right)}.$$

A.7 Proof of Lemma 3: the risk premium under CSEU with NEO-additive capacities

The CSEU of the gamble in question is given by:

$$CSEU=\gamma u\left(y+k\varepsilon_{n}\right)+\lambda u\left(y+k\varepsilon_{1}\right)+\left(1-\gamma-\lambda\right)\sum_{i=1}^{i=n}q_{i}u\left(y+k\varepsilon_{i}\right).$$

Define the risk-premium $\pi(k;.)$ in the usual manner by $CSEU-u\left(y-\pi(k;.)\right)\equiv 0$. Differentiating the previous identity twice with respect to k yields

$$\begin{aligned} 0 \quad &\equiv\quad \gamma\varepsilon_{n}u'\left(y+k\varepsilon_{n}\right)+\lambda\varepsilon_{1}u'\left(y+k\varepsilon_{1}\right)\\ & \quad +\left(1-\gamma-\lambda\right)\sum_{i=1}^{n}q_{i}\varepsilon_{i}u'\left(y+k\varepsilon_{i}\right)+\pi'(k;.)u'\left(y-\pi(k;.)\right),\\ 0 \quad &\equiv\quad \gamma\varepsilon_{n}^{2}u''\left(y+k\varepsilon_{n}\right)+\lambda\varepsilon_{1}^{2}u''\left(y+k\varepsilon_{1}\right)+\left(1-\gamma-\lambda\right)\sum_{i=1}^{n}q_{i}\varepsilon_{i}^{2}u''\left(y+k\varepsilon_{i}\right)\\ & \quad +\pi''(k;.)u'\left(y-\pi(k;.)\right)-\left[\pi'(k;.)\right]^{2}u''\left(y-\pi(k;.)\right).\end{aligned}$$

Evaluate the initial identity and the two derivatives at $k=0$. This yields:

$$\pi(0;.)=0,\pi'(0;.)=-\left(\gamma\varepsilon_{n}+\lambda\varepsilon_{1}\right),$$

$$\pi''(0;.) = \left[-(\gamma\varepsilon_n + \lambda\varepsilon_1)^2 + \gamma\varepsilon_n^2 + \lambda\varepsilon_1^2 + (1-\gamma-\lambda)\,\sigma_\varepsilon^2\right] A(y).$$

By a second-order MacLaurin expansion around $k=0$, $\pi(k;.) = \pi(0;.) + k\pi'(0;.) + \frac{k^2}{2}\pi''(0;.)$. Substitution then yields:

$$\pi_{CSEU}^{RA}(t;.) = -k(\gamma\varepsilon_n + \lambda\varepsilon_1) + \frac{k^2}{2}\left[\begin{array}{c} \gamma\varepsilon_n^2 + \lambda\varepsilon_1^2 - (\gamma\varepsilon_n + \lambda\varepsilon_1)^2 \\ +(1-\gamma-\lambda)\,\sigma_\varepsilon^2 \end{array}\right] A(y).$$

The premium for eliminating ambiguity alone, for its part, is implicitly defined by $CSEU - \sum_{i=1}^{i=n} q_i u(y + k\varepsilon_i - \pi(k;.)) = 0$. Proceeding as above yields:

$$\pi_{CSEU}^{A}(t;.) = -k(\gamma\varepsilon_n + \lambda\varepsilon_1) + \frac{k^2}{2}\left[\begin{array}{c} (\gamma\varepsilon_n^2 + \lambda\varepsilon_1^2) - (\gamma\varepsilon_n + \lambda\varepsilon_1)^2 \\ -(\gamma+\lambda)\,\sigma_\varepsilon^2 \end{array}\right] A(y).\blacksquare$$

A.8 Proof of Proposition 5: risk premia for the tax compliance gamble under CSEU with NEO-additive capacities

Consider the expression for the welfare of the taxpayer at the optimum, as given in the proof of Proposition 2:

$$\begin{aligned} & CSEU(\gamma,\lambda;z) \\ = \; & u(y) + \frac{u'(y)}{2A(y)} \frac{\left(1-p-p\mu_\theta - p\left[\gamma(\theta_n-\mu_\theta) + \lambda(\theta_1-\mu_\theta)\right]\right)^2}{1-p+p(\mu_\theta^2+\sigma_\theta^2) + p\left[\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right) + \lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)\right]}. \end{aligned} \tag{A.14}$$

The premium φ^{RA} associated with eliminating both risk and ambiguity concerning the value of the penalty rate is implicitly defined by:

$$CSEU(\gamma,\lambda;z) = pu\left(y - \mu_\theta tz - \varphi^{RA}\right) + (1-p)\,u(y + tz - \varphi^{RA}).$$

A second-order Taylor expansion of the RHS of this expression yields:

$$u(y) - u'(y)\,\varphi^{RA} + \frac{1}{2}u''(y)\left(\varphi^{RA}\right)^2 + (1-p-p\mu_\theta)\left(1 + A(y)\varphi^{RA}\right)u'(y)tz + \frac{1}{2}\left(1-p+p\mu_\theta^2\right)u''(y)t^2z^2.$$

Optimizing with respect to z and substituting back into the objective function yields:

$$u(y) + \frac{u'(y)}{2A(y)}\frac{(1-p-p\mu_\theta)^2}{(1-p+p\mu_\theta^2)} + \left[\frac{(1-p-p\mu_\theta)^2}{(1-p+p\mu_\theta^2)} - 1\right]u'(y)\,\varphi^{RA} + \frac{u''(y)}{2}\left[1 - \frac{(1-p-p\mu_\theta)^2}{(1-p+p\mu_\theta^2)}\right]\left(\varphi^{RA}\right)^2.$$

Equating this expression with (14) and solving the ensuing quadratic equation in φ^{RA} yields:

$$\varphi^{RA}(\gamma,\lambda) = -\frac{1}{A(y)}\left(1 \pm \sqrt{\frac{1 - \frac{\left(1-p-p\mu_\theta - p\left[\gamma(\theta_n-\mu_\theta)+\lambda(\theta_1-\mu_\theta)\right]\right)^2}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\left[\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right)+\lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)\right]}}{1 - \frac{(1-p-p\mu_\theta)^2}{1-p+p\mu_\theta^2}}}\right).$$

Now consider premium φ^A for eliminating only ambiguity concerning the penalty rate, while risk remains. This is implicitly defined by

$$CSEU(\gamma,\lambda;z) = p\sum_{i=1}^{i=n} q_i u\left(y - \theta_i tz - \varphi^A\right) + (1-p)\,u(y + tz - \varphi^A).$$

A second-order Taylor expansion of the RHS of the preceding expression yields:

$$u(y) - u'(y)\varphi^A + \frac{1}{2}u''(y)\left(\varphi^A\right)^2 + (1-p-p\mu_\theta)\left(1+A(y)\varphi^A\right)u'(y)\,tz + \frac{1}{2}\left[1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)\right]u''(y)\,t^2z^2.$$

Optimizing with respect to z and substituting back into the objective function yields:

$$u(y)+\frac{u'(y)}{2A(y)}\frac{(1-p-p\mu_\theta)^2}{\left[1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)\right]}+\left[\frac{(1-p-p\mu_\theta)^2}{\left[1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)\right]}-1\right]u'(y)\varphi^A+\frac{u''(y)}{2}\left[1-\frac{(1-p-p\mu_\theta)^2}{\left[1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)\right]}\right]\left(\varphi^A\right)^2.$$

Equating, as before, this expression with (14) and solving for the ensuing quadratic equation in φ^A yields:

$$\varphi^A = -\frac{1}{A(y)}\left(1\pm\sqrt{\frac{1-\frac{(1-p-p\mu_\theta-p[\gamma(\theta_n-\mu_\theta)+\lambda(\theta_1-\mu_\theta)])^2}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)+p\left[\gamma\left(\theta_n^2-\mu_\theta^2-\sigma_\theta^2\right)+\lambda\left(\theta_1^2-\mu_\theta^2-\sigma_\theta^2\right)\right]}}{1-\frac{(1-p-p\mu_\theta)^2}{1-p+p\left(\mu_\theta^2+\sigma_\theta^2\right)}}}\right).\blacksquare$$

A.9 Proof of Proposition 6: comparative statics of the equilibrium under ambiguity (EUA) of the tax-compliance game

The FOC which implicitly defines the optimal penalty rate is given by

$$(1+\mu_\theta)\,tz - \frac{\partial c\left(\xi, p^{EUA}\right)}{\partial p} = 0,$$

while optimal compliance behavior is given by the second-order approximation derived in Proposition 1. Evaluating the preceding FOC at $z = z^*_{CSEU}(p^{EUA},\rho)$ and differentiating with respect to ρ yields:

$$\frac{dp^{EUA}}{d\rho} = -\frac{(1+\mu_\theta)\,t\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial\rho} - \frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p\partial\xi}\frac{d\xi(\theta_1,\theta_n,\rho)}{d\rho}}{(1+\mu_\theta)\,t\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p} - \frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p^2}}.$$

Now consider optimal compliance behavior, which we evalute at $p = p^{EUA}$. Differentiating with respect to ρ yields:

$$\frac{dz}{d\rho}^{EUA} = \frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p}\frac{dp^{EUA}}{d\rho} + \frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial\rho}.$$

Substituting for $\frac{dp^{EUA}}{d\rho}$ from the previous expression and rearranging yields

$$\frac{dz}{d\rho}^{EUA} = \frac{-\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p}\frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p\partial\xi}\left(\frac{d\xi(\theta_1,\theta_n,\rho)}{d\rho} - \frac{\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial\rho}\frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p^2}}{\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p}\frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p\partial\xi}}\right)}{-(1+\mu_\theta)\,t\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p} + \frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p^2}}.$$

This expression, combined with the fact that $-\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p}\frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p\partial\xi} > 0$ and $-(1+\mu_\theta)\,t\frac{\partial z^*_{CSEU}(p^{EUA},\rho)}{\partial p} + \frac{\partial^2 c\left(\xi,p^{EUA}\right)}{\partial p^2} > 0$ proves the Proposition.$\blacksquare$

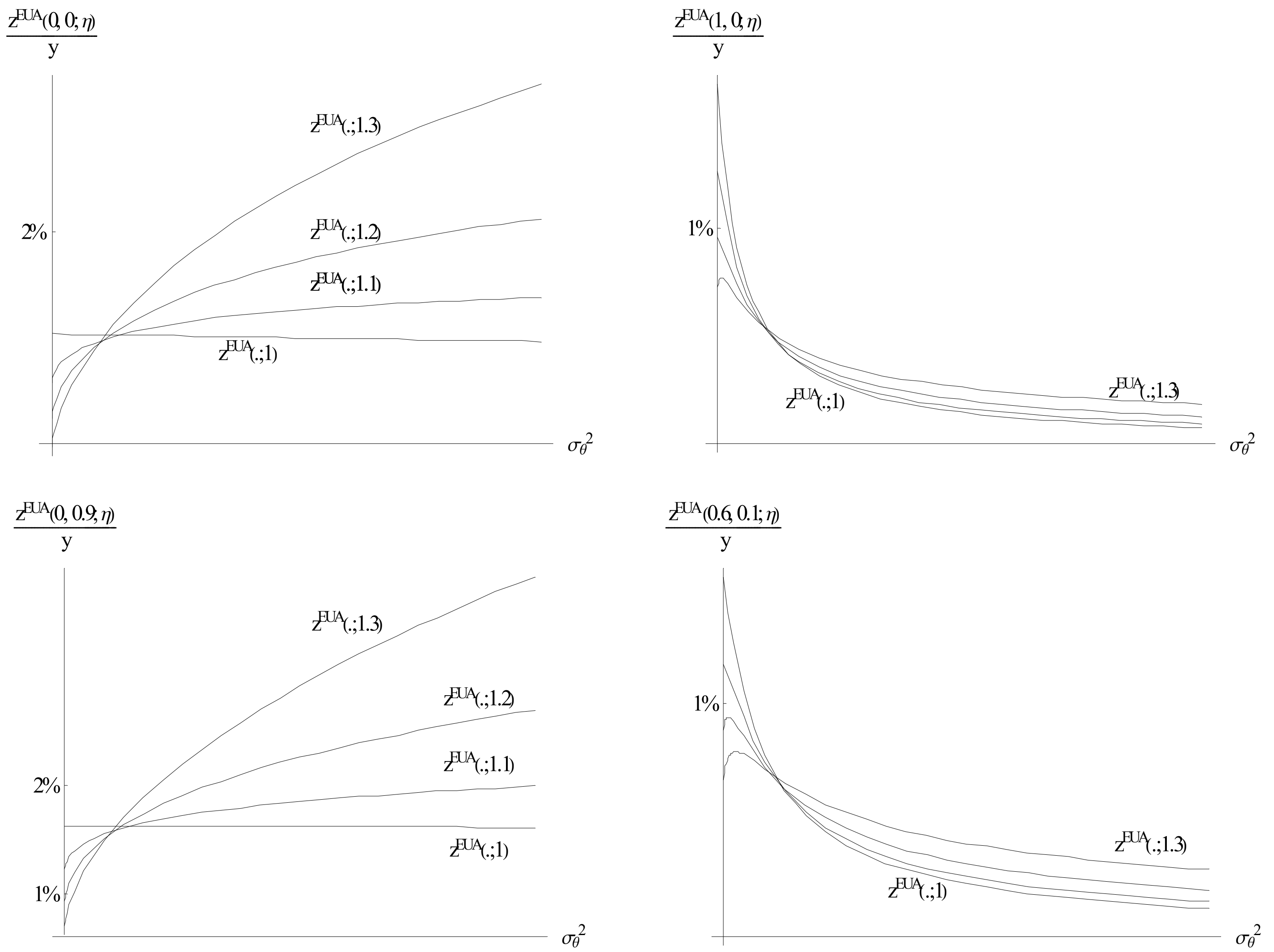

Figure 10: $\frac{z^{EUA}(.)}{y}$ as a function of σ_θ^2 for the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 100$, $t = 0.3$, $p = 0.03$, $c = 10$, $\theta_1 = 0$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 \in [0, 2.5]$.

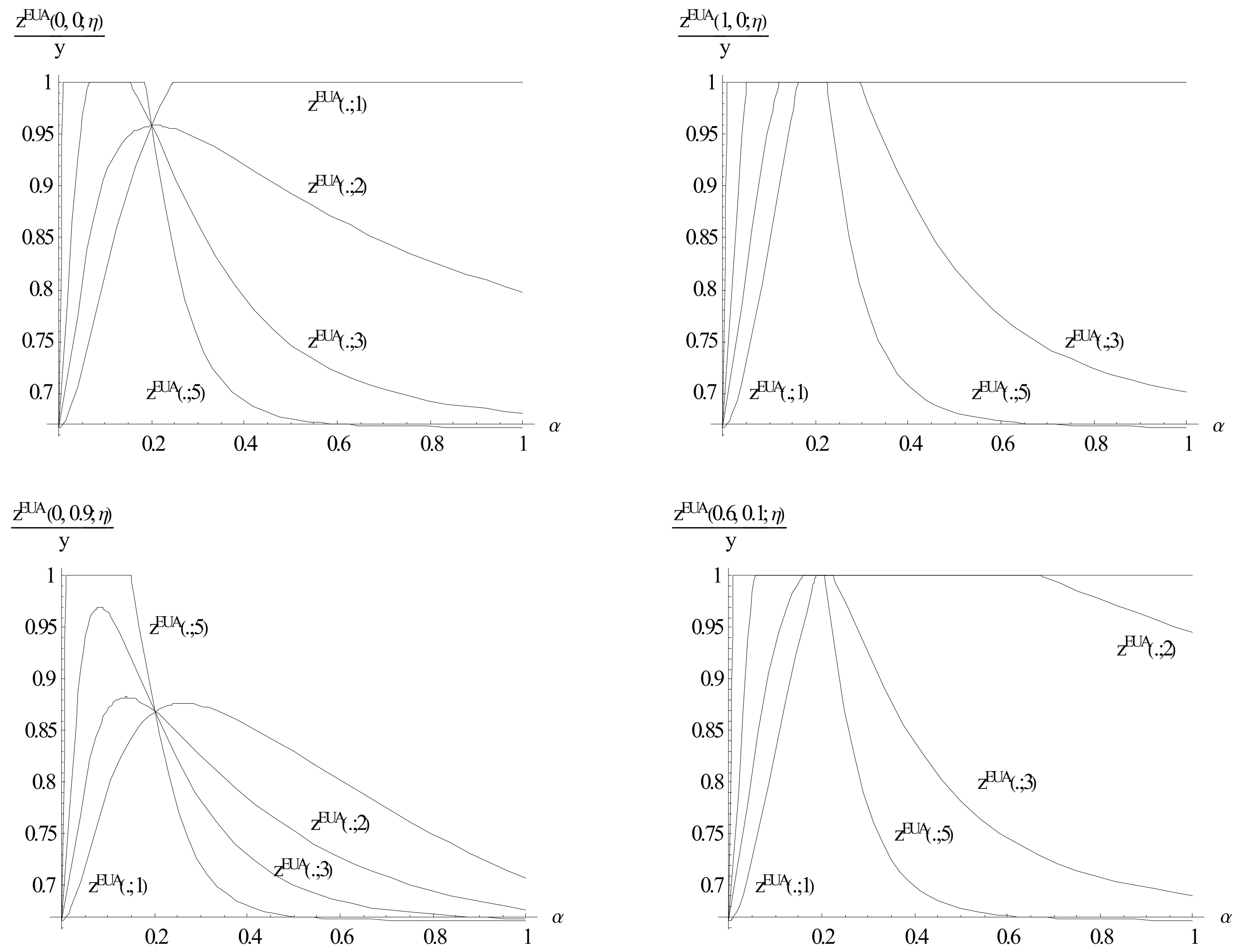

Figure 11: $\frac{z^{EUA}(.)}{y}$ using penalty rates distributed according to the arcsin density and with the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.5$, $y = 10$, $t = 0.3$, $p = 0.03$, $c = 10$, $\theta_1 = 0$, $\theta_n = 3$.

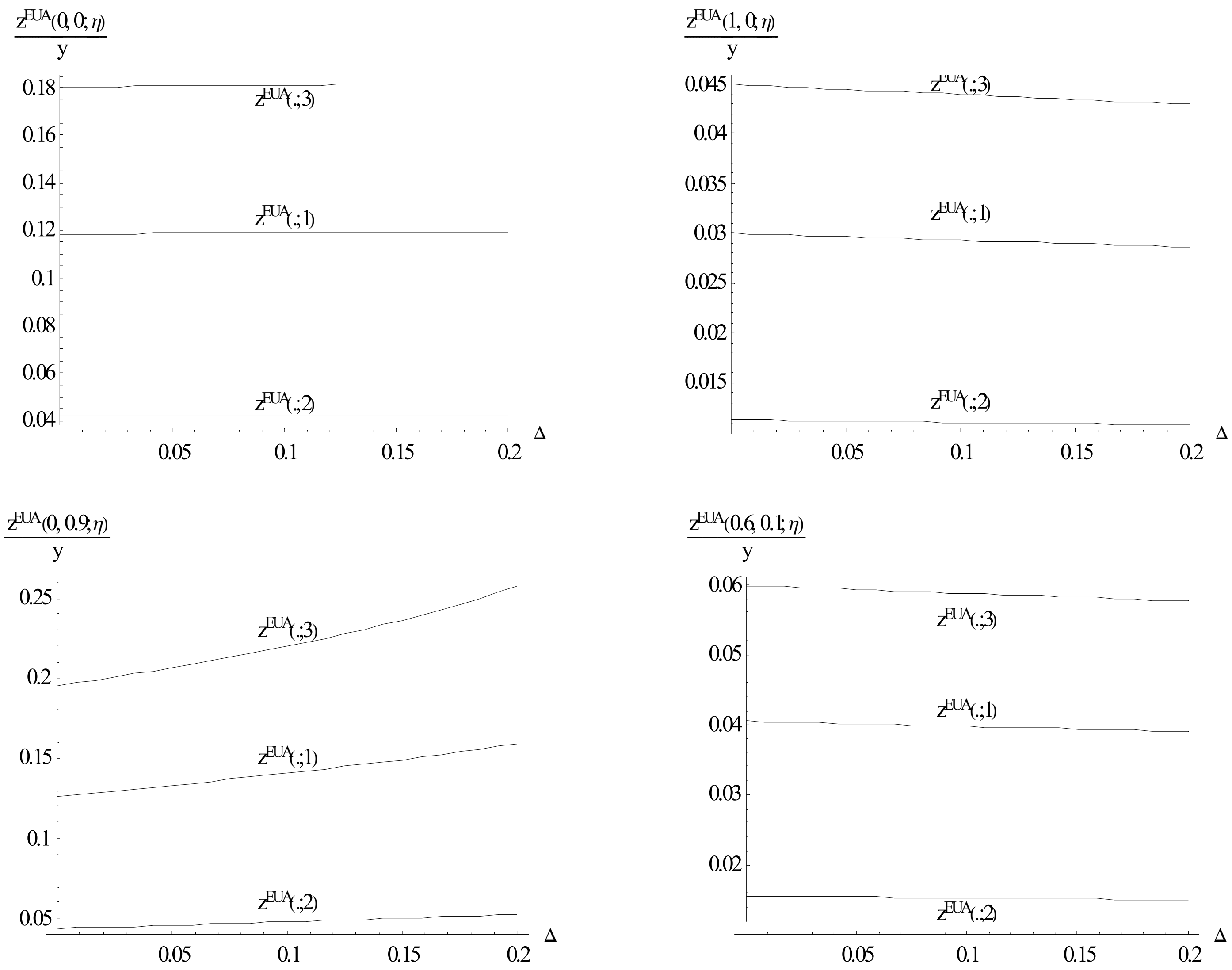

Figure 12: $\frac{z^{EUA}(.)}{y}$ as a function of Δ, with the following parameterization: $u\,(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 10$, $t = 0.3$, $p = 0.03$, $c = 10$, $\theta_1 = 0.2$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 = 0.08$.

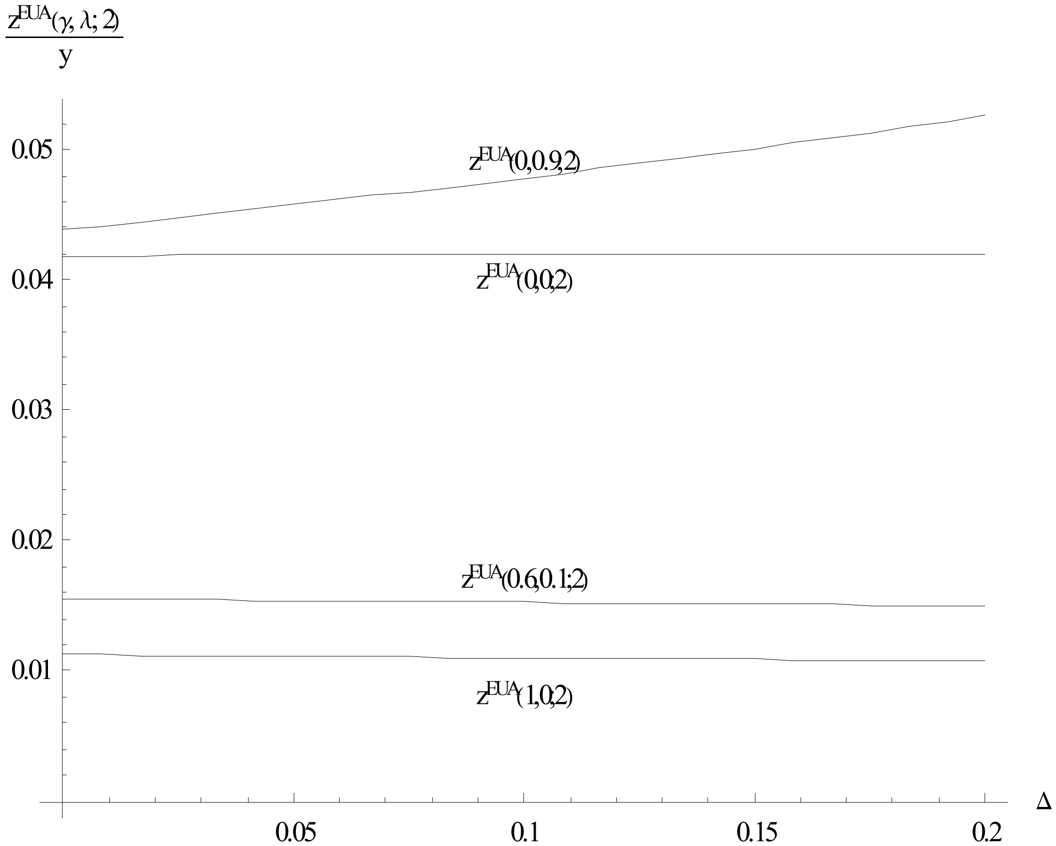

Figure 13: $\frac{z^{EUA}(.)}{y}$ as a function of Δ, with the following parameterization: $u(x) = -\frac{e^{-\nu x}}{\nu}$, $\nu = 0.05$, $y = 10$, $t = 0.3$, $p = 0.03$, $c = 10$, $\theta_1 = 0.2$, $q_1 = 0.5$, $\mu_\theta = 0.25$, $\sigma_\theta^2 = 0.08$.