Cooperation Breakdowns under Incomplete Property Rights*

Jean-Pierre Tranchant†

CERDI - Université d'Auvergne ‡

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Abstract

In order to analyze conflict and cooperation between a State and a non ruling group in a general equilibrium, I unite pure rent-seeking models and economic models of conflict under an assumption of incomplete property rights. I show that a unique and globally stable Nash equilibrium exists in this game. Cooperation breakdowns appear to be twofold: generalized conflict driven by a collapse of the State and one-sided rebellion due to the coexistence between a strong State and a weak minority. Natural resources raise the cost of rebellion and induce the ruler to be more benevolent.

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†Corresponding author. E-mail: j-p.tranchant@u-clermont1.fr; Tel.: +33-4-73-17-74-00; Fax: +33-4-73-17-74-28

‡65 Bd François Mitterand, 63000 Clermont-Ferrand, FRANCE

1 Introduction

Most of today's wars take place inside countries rather than between them. This helps explain the increased number of both theoretical and empirical economic articles devoted to civil conflict issues. The usual underlying theoretical background of these studies is a rent-seeking game between the State and the rebels (see Grossman (1991), Gershenson and Grossman (2000), Collier (2004) among others). Most of the papers isolate the issue of appropriation from the issue of production. They emphasize only the properties of the appropriation function and the value of rents to explain conflict intensity. Hence, taking into account the asymmetry between the contestants about the distribution of power, gives, in this framework, quite limited results.

In this paper, I explicitly model production decisions jointly with rent seeking ones following studies of Skaperdas (1992), Hirshleifer (1995) or Grossman and Kim (1995). Groups are associated in production and rivals in appropriation and equilibrium level of unproductive expenditures is dependent upon characteristics of both technologies. But, opposite to them, I do not postulate a "state of nature" but rather an incomplete rule of law [1]. This assumption lies at the heart of the model whose purpose is to apply the notion of incomplete property rights to civil conflict determination. The idea is the following States only have a partial control of their jurisdictions because they are i) either in formation , ii) closed to collapse or iii) simply too weak to maintain the public order everywhere (Herbst (2000)). Incompleteness of the rule of law implies that in one part of the territory the State is strong enough to enforce the property rights while in the remaining part, anarchy prevails. Examples of such a configuration can be found in Colombia (where FARC[2] controls a part of the country), in

[1] The work which is closer to mine is Skaperdas and Syropoulos (1998) where a prize is introduced in a production function as a fixed factor. But they posit also that revenues are distributed only along the balance of forces.

[2] The Revolutionary Armed Forces of Colombia.

Russia (with Chechnya) or in Sudan. The partition of those countries between the rebels and the State is generally seen as the result of the civil war rather than the effect of an incomplete enforcement of the rule of law. However, the key assumption of the model is not that there exists necessarily a conflict in the anarchic environment but that such a conflict is possible. The major concern of the paper is twofold: to understand the causes of cooperation breakdowns and to characterize the behavior of the State under this environment given that those two issues are closely linked.

That is why this article lies also within the scope of the economic literature of the State. Acemoglu (2004, 2005) , Grossman and Noh (1990) or McGuire and Olson (1996) model the behavior of a self-interested and rational ruling elite. They assume that the ruler maximizes his revenue from taxation subject to a constraint of exit, revolution, or discouragement of the investment from the population. In this paper, the assumption of an absolute ability to tax the citizens for the "stationary bandit" is restrained to a part of the territory, where property rights are enforced. Elsewhere, the State cannot tax nor control the resources which are therefore contestable. To put it differently, I replace the conventional constraints by a constraint of rebellion where the State is not sufficiently strong.

Causes of cooperation breakdowns lies in the strength of the State (i.e. the degree of enforcement of the property rights) which is the crucial determinant of the behavior of the State (benevolent or predatory) which is itself a crucial determinant of the behavior of the non ruling groups. Throughout the article, the strength of the State remains exogenous and I focus on its consequences: the level of cooperation and conflict. The notion of strength of the State is twofold in the model: it refers to the ability to use force and to the ability to enforce the rule of law. However, the importance of the rule of law may be closely linked with State's military force and fairness of the legal system. Hence, the model could exhibit fuzzy situations which could be ruled out. Nonetheless, the special case of a coexistence between low military skills for the State and widely enforced property rights deserves to be studied. Though this appears to be clearly an incongruity for both the contractualist theory of the State and the economic one (Kai and Skaperdas (1999), Skaperdas (2002), Moselle and Polak (2001), Muthoo (2004) among others). The two approaches agree with the postulate that the State must be firstly the most powerful player in the state of nature before to progressively become a genuine State. However, the formation of most of the States in the developing world is not the fruit of a progressive transition from a Lord to a State but rather a leg from former colonialist powers. Hence, the case of a weak State which has inherited a genuine enforcement of the property rights will not be rejected in the paper [3].

Where the institutions prevail, the State and the minority cooperate for producing through a joint production function and the distribution of the output is summarized by a parameter α. Until the last section of the paper, this parameter is considered as exogenous to emphasize the patterns of inter-groups competition. Where the property rights are absent, groups can compete to control the resources of this area. Groups have two strategies to their disposal (produce or fight) and the resolution of the game is the Nash equilibrium. The level of conflict appears to be growing with the weakness of the State and the value of the contestable rents and decreasing with the efficiency of the production process. Then, the endogenous choice of the level of distribution (or equivalently, the rate of taxation) by the ruler is analyzed. Once again, the strength of the State is the key determinant of the kind of politics led by the stationary bandit. A strong State is more predatory than the weak ones, a result close

[3] It may be argued that is only a transitory configuration leading to the collapse of the State. The study of such a State is nevertheless interesting since it fits well, to my mind, the reality of African politics after the decolonization.

to those of Kai and Skaperdas (1999), Acemoglu (2004) or Moselle and Polak (2001) although my model is purely of a static form while their works deal with a dynamic setting. Moreover, the temptation for a State to be predatory is growing with its strength and the weakness of the minority. Although the model confirms that the degree of general conflict is increasing with State failure , the rebellion (defined as the specialization of the non ruling group in conflict activity) is the result of the coexistence between a strong State and a weak minority. Hence, while Somalia's chaos and Chechnya's secessionist war are merged into the general classification of civil war, the model suggests that these two situations correspond to dramatically distinct patterns of weakness and strength of the contestants.

The next section presents the model, the third one develops the analysis of the Nash equilibrium. The fourth part emphasizes the conditions of uniqueness and comparative statics and the fifth section of the paper is devoted to the first stage of a static game in extensive form where a ruler can use distribution as a strategy. Part six concludes.

2 The Model

The most part of the following assumptions about the choice between production and appropriation are based on Skaperdas (1992). Let two groups indexed, by $i = 1, 2$, be associated in a joint production function, f. They are endowed by a similar amount of available resources normalized to one[4] . They can allocate it to production (y_i) or to appropriation (x_i), with $1 = y_i + x_i$. The expenditures in conflict are socially wasteful since the contest for resources is a zero-sum game and they are diverted from the production process. f is increasing with

[4]Like Skaperdas (1992), I posit that the two players have an equal rate of transformation of their endowments to arms or inputs (one to one).

the inputs at a decreasing marginal rate. The share of the divisible rent that resorts to player 1 is determined by the contest success function, p, ($(1-p)$ for player 2), and the players face decreasing marginal returns. The share of output for player 1 (2) is equal to $\alpha \in [0,1]$ $(1-\alpha)$, the value of contestable resources is T and the parameter $\beta \in [0,1]$ reflects the part of the contestable resources in the economy, i.e. the strength of the State. The players are both risk neutrals and maximize their revenues. The payoffs are:

$$\pi^1 = p(x_1,x_2)\beta T + \alpha(1-\beta)f(1-x_1,1-x_2) \quad (1)$$

$$\pi^2 = (1-p(x_1,x_2))\beta T + (1-\alpha)(1-\beta)f(1-x_1,1-x_2) \quad (2)$$

Note that when $\beta = 1$, i.e. in absence of property rights, the game is similar to rent-seeking contests[5]. The fact that the players cannot produce, even separately, from the resources they may extract is clearly a simplification. However, literature on civil conflict emphasizes the negative role played by the natural resources. Moreover, the production issued from those areas is generally no more than an extraction and a sell of primary commodities (opium poppy in Afghanistan, cocaine in Colombia, diamonds in Sierra Leone or timber in Liberia for examples). This simplification is then not unacceptable.

2.1 The production technology

ASSUMPTION 1

$f_i < 0, \forall i = 1,2$

$f_{ii} < 0, \forall i = 1,2$

$f_{ij} > 0, \forall i,j$ with $i \neq j$

$f(0,y_2) = f(y_1,0) = 0$

[5]For a survey of rent-seeking literature, see Nitzan (1994)

$\lim_{x_i \to 1} f_i \to -\infty$
$\lim_{x_i \to 0} f_i \to 0$
f exhibits constant returns of scale.

The hypotheses on the production function are classical except the fact that f_i does not measures the marginal productivity of y_i but its opposite (the marginal loss of production following a reallocation of endowments to appropriation). f_{ii} is negative since the decreasing marginal return law makes each additional unit of endowment diverted from production potentially more effective as we get closer to 0 (in inputs terms). The second cross derivative in respect to x_1 and x_2 is positive, involving that the marginal productivity of one group is positively affected by the raise of the productive effort of the other one, taking into account the positive spillover effect of war (or peace). On the other hand, the more player expends in the contest, the higher the opportunity cost of a unit diverted from production to appropriation. While the former assertion refers to the strategic effect of productive expenditures, the latter refers to direct effect[6].

Inada conditions in addition to the property of nullity of the output when one input is null are consistent with Cobb-Douglas function. The last assumption prohibits asymmetric equilibria for which one player does not produce. Indeed, in that case, the best response for the other player is to produce nothing too.

2.2 The conflict technology

The conflict technology (or *contest success function*, CSF) relates arms investment to the share of rents won, as I postulate resources are divisible. Such functions have been introduced in many areas of economics and take generally a ratio form, as Tullock (1980) presented it. Then, the probability of success for player i is $p^i = \frac{h_i(x_i)}{\sum_i h_i(x_i)}$. However, this functional form suffers from a great

[6]Roughly speaking, the stability of the game depends on the property that the former ones (included strategic effect in the contest) are dominated by the latter ones. This point is extensively discussed in section 4.1

drawback for my model since p is not defined for zero level expenditures by all players as well as the partial derivatives of p in respect to the effort when all efforts are initially null. Adding a noise in the ratio-form CSF allows continuity of first and second derivatives even when one or both efforts are null, a property obviously desirable in warfare context. Amegashie (2005) proposes the following function where p_i is the probability of win for player i and N the number of contestants: $p^i = \frac{h_i(x_i)+d}{\sum_i h_i(x_i)+Nd}$[7]. The parameter d can be seen as a "luck factor", since the more important d, the less decisive the balance of forces. I will rather refer to d as a country fixed effect reflecting the national cover of forests, mountains or rough terrain which weakens the importance of asymmetries in technologies and balance of forces among players [8]. If each player faces the same conflict technology, all their partial derivatives have same absolute values and opposite signs. Following assumptions are consistent with the generalized ratio-form function for a non null noise.

ASSUMPTION 2:

$\infty > p_1 > 0$

$-\infty < p_2 < 0$

$-\infty < p_{11} < 0$

$\infty > p_{22} > 0$

$p_{12} \gtreqless 0$ as $h_1(x_1) \gtreqless h_2(x_2)$

$p(0,0) = 1/2$

The CSF is concave in own players's effort for all level of x_i and second-cross partial derivatives is positive for player i only if $h_i(x_i) > h_j(x_j)$.

[7]Note that Hirshleifer (1989) proposed a alternative specification of the CSF based on the difference between efforts rather than ratio. For an axiomatic discussion on CSF, see Skaperdas (1996) and Clarke and Riis (1998).

[8]The fixed effect interpretation permits to avoid the specification of a distribution law of the luck. Rough terrain is recognized as a factor favoring the less skilled or numerous contestant (Fearon and Laitin (1999)), exactly as d plays in the CSF. In addition, the presence of a noise is justified by a tractability demand, however, it is not a critical assumption for the results.

3 Cooperation or Conflict?

Each player maximizes this program:

$$x_i^* = \arg\max_{\{x_i\}} \pi^i(x_i) \tag{3}$$

Lemma 1 *Under assumption 1-2, there exists at least one pure Nash Equilibrium*

Proofs of lemma 1 and following results are in appendix.

FOCs give:

$$\pi_1^1 = p_1\beta T + \alpha(1-\beta)f_1 = 0 \tag{4}$$

$$\pi_2^2 = -p_2\beta T + (1-\alpha)(1-\beta)f_2 = 0 \tag{5}$$

SOCs are:

$$\pi_{11}^1 = p_{11}\beta T + \alpha(1-\beta)f_{11} < 0 \tag{6}$$

$$\pi_{22}^2 = -p_{22}\beta T + (1-\alpha)(1-\beta)f_{22} < 0 \tag{7}$$

The first part of the right hand side of 4 and 5 constitutes the marginal benefit of an additional unit of endowment devoted to contest whereas the second part constitutes its marginal cost. Obviously, an interior equilibrium is characterized by the equalization of the marginal cost and the marginal benefit. The concavity of p and the convexity of f ensure that SOCs are always satisfied.

Throughout the paper, I will refer to the game as a symmetric game in the sense of the assumption 3 below:

ASSUMPTION 3

Players are symmetric so that $\left|\frac{f_i}{p_i}\right| = \left|\frac{f_j}{p_j}\right|, \forall x_i = x_j$

An asymmetric setting of the game is provided by the assumption 4 below:

ASSUMPTION 4

Consider an asymmetric distribution of skills so that one player enjoys more relative ability in production than his opponent:

$\left|\frac{f_i}{p_i}\right| > \left|\frac{f_j}{p_j}\right|, \forall x_i = x_j$

Assumption 4 implies either that player i is more efficient in production than his opponent while they face similar technology of appropriation, either the two players are symmetric in production and player i is less efficient in appropriation than player j. I will focus on the first interpretation.

Although the primary goal of the paper is to focus on the asymmetry of power, I will investigate the introduction of another source of asymmetry like in assumption 4 when it is insightful.

3.1 Interior Equilibrium

Proposition 1 *The Nash equilibrium is interior if $\beta \in]0,1[$ and if $T \neq 0$*

Given assumption 1-2, if a player allocates all his endowment to the contest, the marginal cost of x_i is infinitely negative. On another hand, if he rather specializes absolutely in production, the marginal benefit of y_i is null. Since p_i is always positive and finite by assumption 2, a corner solution will emerge only if the player has no choice about activity, i.e. if β is at a bound and/or if T is null. If those conditions are not fulfilled, the Nash equilibrium is necessarily interior. Throughout the paper, I will focus on interior solution.

Proposition 2 *The pure Nash equilibrium is characterized by the following properties:*

i

$$-\alpha \frac{f_1^*}{p_1^*} = (1-\alpha)\frac{f_2^*}{p_2^*}$$

ii

$$\begin{aligned}
\alpha &= -\frac{p_1^*}{f_1^*}\frac{\beta}{(1-\beta)}T \\
(1-\alpha) &= \frac{p_2^*}{f_2^*}\frac{\beta}{(1-\beta)}T
\end{aligned}$$

iii

$$\frac{\beta}{(1-\beta)}T = \left|\frac{f_1^*}{p_1^*}\right| + \left|\frac{f_2^*}{p_2^*}\right|$$

Under assumption 3, if α is equal to 1/2, then $x_1^* = x_2^*$ and interior equilibrium is symmetric.

Under assumption 4, when $x_1^* = x_2^*$, the more skilled player must receive a lower share of rents than his rival. For $\alpha = 1/2$, he must contribute more in production than his opponent.

Recall that, under assumption 1-2, both $|f_i|$ and $|\ p_i|$ monotonically increase with x_i. Since f is convex and p concave in x, the ratio $\left|\frac{f_i}{p_i}\right|$ is monotonically increasing with x_i. Then, if $\alpha = 1/2$, the more efficient player will contribute more in production at interior equilibrium than his opponent. Conversely, players expend the same amount of their endowments in production only if the most efficient among them receives a lower share of the output than the other player. To put it differently, when one player is relatively more efficient in production, he faces a greater opportunity cost of arms. Thus, if the distribution of output is egalitarian, he contributes more in production at the equilibrium so that the ratio $\left|\frac{f_i}{p_i}\right|$ is the same for the two contestants. If the distribution is skewed in favor of the "better" player, the asymmetry of equilibrium expenditures is enhanced relative to the situation of equality of the distribution. In contrast, when the distribution is unfavorable to the most efficient producer, the asymmetry is limited. In economic models of conflict, the most efficient producer contributes more to production too, but he obtains the lowest share of the output. Indeed, the distribution is equal to the ratio of arms expenditures[9]. In this present model, however, there is no link between allocation choices and distribution, which is purely exogenous.

The originality of the previous propositions is that they link simultaneously properties of technologies, distribution of output and value of contestable rents. Indeed, economic models of conflict emphasize only technologies while rent-seeking models focus on properties of contest success function and on value of rents. Since the model append to the canonical rent-seeking framework a joint production function, key findings include conditions over a significantly greater number of parameters and technologies properties than each of two pioneering classes of models separately. The aim is to give a more comprehensive and adapted framework to understand civil conflict patterns.

Part iii of proposition 2 highlights that the nature of equilibrium is crucially dependent upon the value $\frac{\beta}{(1-\beta)}T$, which is a measure of the potential prize of the appropriation. Nevertheless, for a given value of this prize, the more the production process is efficient and the less the conflict technology is decisive, the less the equilibrium will be conflicting. An outcome predominantly characterized by rivalry is then driven by both a weakness of the State and a relative inefficiency of the production. If we introduce a parameter, A, which reflects the level of institutional quality or the global factor productivity in f, we can see that the less developed countries (where A is low) are far more prone to a civil conflict than the developed one for a given weakness of the State. Near anarchic equilibrium can be seen as closed to the Somalian case since the end of the latter civil war. None group is able to enforce property rights and, as a result,

[9]A conclusion consistent with the assumption of complete absence of rules in those models.

factions fight or negotiate under the threat of open conflict for appropriation of territories. In opposite to this dramatic case, near full cooperation equilibrium is known by countries where property rights are almost perfectly enforced by the State and/or where the economic institutions are growth-promoting. Illustrations can be found in Western world but also in non democratic developing countries like North Korea, Togo under Eyadema presidency or Saudi Arabia. These few examples highlight the non normative approach of the model since the ethic nature of political regime is not integrated in the analysis. Absence of conflict is a Pareto-optimal equilibrium only if we neglects the fact that rebellion, although greed-driven, can occurs under a totalitarian régime whose failure cannot be seen as a bad outcome.

To summarize, the level of asymmetry among the players, the relative efficiency of production and the degree of enforcement of the rule of law lead to very different configurations of the economy. Thus, the interior equilibrium may reflect also patterns of conflict so different like the Russia/Chechnya secessionist war, the civil conflict between Northerners and Southerners in Sudan or the relative peaceful cooperation between Blacks and Whites in South Africa since the end of the apartheid.

4 Properties of Nash equilibrium and comparative statics

The aim of this section is twofold: determine if Nash equilibrium is unique and stable and derive the comparative statics of the parameters at equilibrium point.

4.1 Stability and uniqueness of equilibrium

Theorem 1 *Under assumptions 1-2, there exists a unique and globally stable Nash equilibrium if one of the following statements is satisfied on the whole set of feasible actions:*

$$\beta T \geq \frac{1}{4}\frac{f_{11}}{p_{11}}\frac{1}{1-\alpha}$$
$$\beta T \geq -\frac{1}{4}\frac{f_{22}}{p_{22}}\frac{1}{\alpha}$$

Theorem 1 guarantees that the game has a unique Nash equilibrium and that, whatever the initial point considered on the interval of actions set, x converges to the equilibrium (where $x = x_1.x_2$). The condition for uniqueness and global stability is not too stringent and requires just a minimal value for T, except for $\beta = 1$ and $\beta = 0$. The first case corresponds to pure rent-seeking contest for which local stability and uniqueness has ever been showed while the latter necessitates that $\alpha = 1/2$. Anyway, this case is ruled out by imposing that $\beta \in]0,1[$[10]. The degree of restriction of uniqueness condition depends on the relation between β and $\left|\frac{f_{ii}}{p_{ii}}\right|$. If they move in the same direction, the restriction is almost the same whatever the level of β and it is just necessary to impose a sufficient value of T to safely interpret the model. The sign of the relation between β and $\left|\frac{f_{ii}}{p_{ii}}\right|$ depends on the sign of the comparative statics, $\frac{d\beta}{dx_i}$. I show in the next subsection that an increase in β implies an increase in x_i. As a result the left hand side (LHS) and the right hand side (RHS) of theorem 1 move in same way when β changes.

Theorem 1 exhibits the conditions required for global stability of a game as demonstrated by Rosen (1965), known as the condition of "strict diagonal

[10]In addition, this case is trivial since whatever the environment, the players have no choice to make. A proper analysis would require that players have an exit option like investing in the informal sector, for example.

concavity" of payoffs. Moulin (1986) gives a computational technique to check it which consists to show that the hessian matrix (H) is negative quasi-definite ("'The univalent mapping argument"'). The detailed sketch of the proof is in the appendix. The global stability notion means that, for any initial point x in the whole strategy space, the dynamics of best-responses leads to a convergence of x to the equilibrium point of the game. Global stability is more powerful than local stability and if the former is true, the latter is true also[11]. Indeed, the condition of local stability (the domination of the diagonal of direct effects over the one relative to indirect effects) is necessarily satisfied if H is negative quasi-definite. Then $detH$ is positive and the Implicit Function Theorem can be safely used for comparative statics.

4.2 Best-reply functions and comparative statics

Using implicit differentiation, we can determine the slopes of best-reply functions for interior equilibrium:

$$\frac{\partial x_1^*}{\partial x_2} = -\frac{p_{12}\beta T + (1-\beta)\alpha f_{12}}{p_{11}\beta T + (1-\beta)\alpha f_{11}} \tag{8}$$

$$\frac{\partial x_2^*}{\partial x_1} = -\frac{-p_{12}\beta T + (1-\beta)(1-\alpha) f_{12}}{-p_{22}\beta T + (1-\beta)(1-\alpha) f_{22}} \tag{9}$$

At symmetric interior equilibrium, both slopes are positive since $p_{12} \to 0$, then actions are strategic complements for each player. Note that for the special case of $f_{12} = 0$, the slopes tend to zero. When the equilibrium is asymmetric, the actions can become strategic substitutes for player i if the two following statements are satisfied:

[11]See Moulin (1986) for an extensive treatment of stability issues.

- $$p_{12} < 0$$

- $$\frac{\beta}{(1-\beta)} > -\alpha \frac{f_{12}}{p_{12}}$$

And reciprocally for player 2.

Action i is a strategic substitute for player j only if $h_i(x_i) < h_j(x_j)$ and is more likely if β is closed to one, i.e. in a situation of almost pure rent-seeking contest [12]. For a game of strategic complementarities, an increase in arms spending for one player creates an incentive for the other one to follow him and reciprocally. Indeed, a reduction of the productive effort for one player decreases the marginal productivity of effort for the other one through f_{12}. However, the global convergence property guarantees that this movement does not lead to a spillover effect since the strategic effects are dominated by direct effects.

Proposition 3 *Under assumption 1-2, the comparative statics at interior equilibrium are given by:*

Effect of the strength of the State:

i $\frac{\partial x_i^*}{\partial \beta} > 0$ *if* $\frac{\partial x_i(x_j)}{\partial x_j} \geq 0, \forall i = 1, 2, i \neq j$

ii $\frac{\partial x_1^*}{\partial \beta} > 0$ *if* $\frac{\partial x_1(x_2)}{\partial x_2} < 0$ *as* $\frac{(-\alpha f_1 + T p_1)}{(-1+\alpha) f_2 - T p_2)} \geq \frac{\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2}}{\frac{\partial^2 \pi^2}{\partial x_2^2}}$

iii $\frac{\partial x_2^*}{\partial \beta} > 0$ *if* $\frac{\partial x_2(x_1)}{\partial x_1} < 0$ *as* $\frac{(-1+\alpha) f_2 - T p_2)}{(-\alpha f_1 + T p_1)} \geq \frac{\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2}}{\frac{\partial^2 \pi^1}{\partial x_1^2}}$

Effect of the rents:

i $\frac{\partial x_i^*}{\partial T} > 0$ *if* $\frac{\partial x_i(x_j)}{\partial x_j} \geq 0$

[12] When the productive part of the economy is more important, best-response functions are increasing, even for the least aggressive player, since production is a cooperative activity in the model.

ii $\frac{\partial x_i^*}{\partial T} > 0$ *if* $\frac{\partial x_i(x_j)}{\partial x_j} < 0$ *as* $-\frac{p_i}{p_j} \geq -\frac{\frac{\partial^2 \pi^i}{\partial x_i \partial x_j}}{\frac{\partial^2 \pi^j}{\partial x_j^2}}, \forall i = 1, 2, i \neq j$

Effect of the distribution:

i $\frac{\partial x_1^*}{\partial \alpha} < 0$ *if* $\frac{\partial x_1(x_2)}{\partial x_2} \leq 0$

ii $\frac{\partial x_1^*}{\partial \alpha} < 0$ *if* $\frac{\partial x_1(x_2)}{\partial x_2} > 0$ *as* $\frac{f_1}{f_2} \geq -\frac{\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2}}{\frac{\partial^2 \pi^2}{\partial x_2^2}}$

iii $\frac{\partial x_2^*}{\partial \alpha} > 0$ *if* $\frac{\partial x_2(x_1)}{\partial x_1} \leq 0$

iv $\frac{\partial x_2^*}{\partial \alpha} > 0$ *if* $\frac{\partial x_2(x_1)}{\partial x_1} > 0$ *as* $\frac{f_2}{f_1} \geq -\frac{\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2}}{\frac{\partial^2 \pi^1}{\partial x_1^2}}$

Proposition 3 states that for a game of strategic complementarities, like at symmetric equilibrium, a raise of T or β leads to an increase of the equilibrium conflict expenditures while the effect of $d\alpha$ remains ambiguous. A collapse of the rule of law or an increase in T (a discovery of oil in anarchic territory, for example) pushes the equilibrium to a more conflicting level. Direct impact of $d\beta$ is twofold: it increases the marginal benefit of arms and it reduces their marginal cost while dT just increases the marginal benefit. In addition to this direct impact, marginal reaction depends also on the strategic effect passing through the best-reply function. As both best-response functions are non decreasing at symmetric equilibrium, the direct effect is reinforced by the strategic effect. Then, the total effect for $d\beta$ and dT is unambiguously positive. However, a change in the distribution creates an opposite direct impact for each player and both are counterbalanced by strategic effect. The sign of the comparative statics is determined by the comparison of direct and strategic effect. If players are perfectly symmetric, f_1/f_2 is equal to one while $-\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2} / \frac{\partial^2 \pi^2}{\partial x_2^2}$ is lower than one given that $p_{12} \to 0$ and $f_{11} = f_{12}$. Hence, a change in the distribution raises the equilibrium level effort of the disadvantaged group whereas it reduces the intensity of fighting for the other one. Marginal comparative statics of symmetric equilibrium is then very intuitive and corroborates both Collier's thesis and Gurr's one about the effect of natural resources and discrimination on intensity of conflict.

Proposition 4 *At symmetric interior equilibrium, the conflict expenditures for each player increase with the amount of rents, the weakness of the State and a lower share of the output.*

Proposition 3 reveals that, for asymmetric equilibrium, the results summarized in proposition 4 do not necessarily hold any more. Comparative statics of dT and $d\beta$ are likely to be reversed for the more peaceful player when an asymmetry over the distribution is introduced. Concerning the effect of $d\alpha$, it is for the less aggressive player that an ambiguity occurs when asymmetry is introduced.

In order to know if for an asymmetric equilibrium, the proposition 3 holds also, a solution is to differentiate each condition contained in proposition 3 with respect to its parameter. Such a method does not permit to totally overcome the ambiguity about the impact of $d\beta$ and dT but permits to assert that an increase in α causes always an increase in x_2 (decrease in x_1) and reciprocally. To do that, it is necessarily to assume that p and f are thrice continuous differentiable functions as well as the following point.

ASSUMPTION 5

i $p_{112} \geq 0$ if $2x_2 + d \geq x_1$

ii $p_{221} \geq 0$ if $2x_1 + d \geq x_2$

iii $f_{iii} > 0, \forall i = 1, 2$

iv $f_{iij} < 0$, $f_{jji} < 0, \forall i = 1, 2$

Assumption 5 is consistent with a ratio-form CSF with a non null noise d and with a Cobb-Douglas production function.

Lemma 2 *Assume that p and f are thrice continuous differentiable functions, and assumptions 1-2-3,5 are verified.*

A greater level of economic discrimination involves an increase in conflict expenditures for the player considered, whatever the initial position of the equilibrium point.

Lemma 2 is a fundamental result for the next section, devoted to the redistribution choice of the ruler.

Collier and the World Bank (Collier (1998), Collier et al. (2003), Collier and Hoeffler (2004)) suggest that the dependence on valuable natural resources is conducive to higher risk of civil war. However, the impact is empirically differentiated as the country is rich or poor. The former ones do not experience high levels of risk if they are endowed by a significant amount of natural resources in opposite to the latter ones. Proposition 3 shows that a higher level of natural resources (T) leads to a higher investment in rent-seeking for both contestants, which is consistent with the empirical findings of Collier. However, this relation is conditional on an incomplete enforcement of property rights, since the rent-seeking is not an option in the opposite case. Thus, rich countries like Norway or Australia, which are high exporters of primary commodities, are not prone to the threat of civil war. Indeed, they have strong States able to control their whole territory, including their natural endowments. Actually, this simple explanation stressing the role of the State ability to enforce its law for preventing the war was already suggested by Fearon and Laitin (2003). The payoffs functions in this model are consistent with this idea, and logically, the predictions of the model emphasize the negative role of natural resources, but conditional on the degree of enforcement of the property rights.

5 Redistribution Choice of the Ruler

An interesting issue concerns the use of a discretionary power over distribution by a ruler, materialized here by a manipulation of α. Azam (1995) has analyzed, in an isolated pure rent-seeking framework, the incentives for the State to redistribute temporary money to the rebels and the patterns of peace and conflict that resulted from this mechanism. He focused on Africa since existence of a discretionary power over categorical distribution is supposed to be more realistic in this continent where weak institutions prevail. However, his analysis suffers from the same drawback that any analysis in partial equilibrium applied to civil conflict issue: it ignores the productive interactions between the State and the potential rebels. The aim of this section is to endogeneize the distributive choice of the ruler in a general equilibrium analysis. To address this issue, I transform the simultaneous game of the previous sections in a two stages game in extensive form. In the first stage, the ruler is the only one to play and he uses the distribution as a strategy, given the best-reply functions determined in the second stage as in previous sections. The second stage is similar to the game of the section 2. Backward induction gives the subgame perfect equilibrium of this game. In addition, there is no time-consistency problem since the ruler applies the tax rate before the non ruling group plays.

Throughout this section, assumption 3 (identical skills among the players) applies since lemma 2 about comparative statics of $d\alpha$ does not necessary hold under assumption 4 (asymmetry). In addition, I will sometimes refer, with a slight abuse of language, to the non ruling group as a "minority" although both players have the same initial endowments and skills [13]. Let denote the State by the superscript "S" and the non ruling group by "R".

When CPOs are satisfied for both players, it is possible to resort to compar-

[13]The term minority means in this context the political submission of the group.

ative statics, the optimization program for the ruler is:

$$\alpha^* = \arg\max \ \pi^S(x_R^*, x_S^*) \tag{10}$$

Then,

$$\frac{d\pi^S(x_R^*, x_S^*)}{d\alpha} = \frac{\partial \pi^S}{\partial \alpha} + \frac{dx_S^*}{d\alpha}\frac{\partial \pi^S}{\partial x_S^*} + \frac{dx_R^*}{d\alpha}\frac{\partial \pi^S}{\partial x_R^*} \tag{11}$$

where:

$\frac{\partial \pi^S}{\partial \alpha} = -(1-\beta)f$

$\frac{\partial \pi^S}{\partial x_R^*} = -\beta T p_1 + (1-\beta)(1-\alpha)f_1$

Obviously, the optimal distribution rate for the ruler is implicitly defined when equation 11 equals zero. By the envelop theorem, the second term of RHS is null and we know that $\frac{\partial \pi^S}{\partial x_R^*} < 0$. Hence, if $\frac{dx_R^*}{d\alpha} > 0$, the ruler is induced to discriminate the minority since a greater level of redistribution would increase the detrimental arms spending level of the latter. But, we know from lemma 2 that it is never true. In contrast, following a raise of the redistribution, there exists a trade-off between the direct loss for the ruler, $\frac{\partial \pi^S}{\partial \alpha}$, and the indirect benefit, $\frac{dx_R^*}{d\alpha}\frac{\partial \pi^S}{\partial x_R^*}$, provided by the reduction of the rebellion level.

Two key elements can be drawn from 11:

i the lower β and the higher f, the greater the direct cost of redistribution for the ruler

ii the more effective the minority is at producing and appropriating, the more the ruler is incited to redistribute.

The first statement reveals that, "*ceteris paribus*", the appropriative behavior of the State is more pregnant if it is strong and the country is rich. This is clearly counter-intuitive and deserves an explanation. First, we can see from 11 that the direct loss of revenue following the redistribution is an increasing function of β and f, the measure of the importance of the production in the economy. Hence, the higher the rule of law, the more important the direct effect. Second, β gets also through the strategic effect and this one could alter the previous statement. To understand this point, it is useful to advance until the next proposition before to go back.

The second statement relates the level of taxation to the level of deterrence caused by the threat of arms investment from the minority. Intuitively, when the cost of such a strategy is high for the ruler, i.e. when the minority is efficient in both technologies, the State should not take too large share of the output.

This second point is closed to the seminal work of Buchanan and Faith (1987) which emphasizes the threat of secession from the minority as an incentive for the ruler to limit the level of taxation imposed on the non ruling group. Generally speaking, the analysis of the redistribution choice proceeded in this section is based on a large literature in economics including Grossman and Noh (1990), McGuire and Olson (1996), Acemoglu (2005), Acemoglu and Robinson (2001), Moselle and Polak (2001) or Kai and Skaperdas (1999) among others. In all cases, the analysis of the political outcome of a self-interested and rational ruling elite is undertaken knowing that this elite faces a constraint from the minority. Buchanan points the threat of secession, McGuire and Olson the discouragement of the investments, Acemoglu and Robinson and Grossman and Noh, the limitation of the survival of the régime following a too unpopular tax rate. The aim of the papers is to seek the conditions of emergence of a benevolent or predatory attitude of the State. In this section, I introduce a new kind of constraint for the State, namely, the threat of rebellion.

As a formal inquiry of equation 11 is cumbersome and too harsh to interpret, I simply analyze a discrete change in the level of redistribution. Let $\alpha^P < \alpha^B$ (The superscript B refers to "benevolent" whereas P means "predatory").

Proposition 5 *Under assumption 1-3,5 and from lemma 2:*

α^P is preferred to α^B, where $\alpha^P < \alpha^B$, if:

Necessary condition $(1-\alpha^B)f^B - (1-\alpha^P)f^P \leq 0$

Extensive-form condition $\frac{\beta}{(1-\beta)}T \leq -\frac{(1-\alpha^B)f^B-(1-\alpha^P)f^P}{p^P-p^B}$

The necessary condition comes from the fact that, when the ruler raises the tax rate, he reduces his effort in the contest whereas the minority does the opposite. He then looses revenues issued from the appropriation and it is therefore essential that he compensates this loss by an increase of his production revenues. Hence, high level of taxation is an event only if the marginal rate of substitution between the inputs of the groups is low so that the necessary condition is satisfied. There exists then a force which limits the level of taxation since the profitability of such a politics implies that the State could easily substitute his own inputs with those of the minority. But decreasing marginal returns in production imply that the marginal productivity of the discriminated group is greater than the one of the elite. This discrepancy raises with the level of taxation. Thus, there exists an upper bound to the level of taxes than the ruler will rationally implement. This upper bound is lower the more efficient is the production process. As a consequence, we should observe less predatory States in countries where A is high, i.e. in the performing economies [14].

Following the adoption of a greater level of taxation we observe a loss of exogenous resources for the ruler and this loss is equal to $\beta T(p^P - p^B)$, i.e. the additional share of rents appropriated by the minority times the value of these rents. It is obvious that the more efficient the warfare technology, the higher the loss for the ruler. In addition, presence of valuable resources must be positively

[14] More rigorously, it is when the marginal productivity is high that the discrimination will be unlikely. Then, emerging countries should be less characterized by predation than rich ones given the assumption of constant returns of scale (and for a discretionary power of the State on the rate of distribution).

linked with redistribution. The last point contradicts the general agreement about the negative correlation between rents and redistribution. However, the argument advanced is that the State extracts high revenues from rents and it is then not induced to fairly redistribute the national income in order to build growth-promoting institutions. But, if such rents are not controlled by the ruler but rather are contestable, it is not unsurprising that the State be more benevolent in order to appease the non ruling group and preserve for himself a sufficient share of the natural resources [15].

Now it is possible to return to the statement i which I only briefly evocate earlier in the paper. We saw the direct and negative effect of the strength of the State on the level of tax rate but what about its strategic effect? Other things being equal, a high β is associated with low productive efforts, then with high marginal productivity. A weak State faces therefore a more stringent necessary condition than a strong State. On an other hand, when β is low, the marginal capacity of appropriation is high, involving that a rebellion is more detrimental for a strong ruler than for a weak one. To sum up, a weak State faces more difficulty to fulfill the necessary condition of discrimination. But as long as this necessary condition is satisfied, a rebellion is less detrimental for a weak State than for a strong State. To put it differently, a low enforcement of property rights exposes more the State to the rebellion threat whereas a rebellion is more detrimental for a strong State, especially if natural endowments are important.

Proposition 6 *Under assumption 1-3 and lemma 2:*

i *The upper limit on taxation is increasing with the strength of the State*

ii *The level of taxation is decreasing with the amount of natural resources*

[15] However, if we posit that rents are associated with low sophisticated production process, the productive complementarities (f_{12}) are probably weak reducing the cost of a rebellion for the ruler. This assumption mitigates the redistributive role of natural resources.

iii *The level of taxation is decreasing with the efficiency of appropriation technology*

iv *The level of taxation is decreasing with economic efficiency*

It must be kept in mind that the model focuses on discretionary power and such a power is plausibly correlated with the degree of democracy of the institutions. Then, the archetype of the predatory State is a strong autocracy. As the dissolution of the rule of law cannot obviously be recommended, the political answer against the excessive predation is democratization. Indeed, such political systems are desirably characterized by the presence of checks and balances in order to prevent any kinds of categorical discrimination. The prediction that redistribution will be lower in strong States than in weak States for non democratic countries was already supported by DeLong and Schleifer (1993) who suggest, following an empirical study on the period 1000-1800 in Europe, that strong absolutist régimes were more predatory than the weaker ones.

Part i of proposition 6 joins up with Acemoglu (2004), Moselle and Polak (2001), Kai and Skaperdas (1999) for examples, who find also that taxation is at its highest level when the State is strong. However, their definitions of the strength of the State are heterogeneous and different from the mine. Note that Acemoglu (2004) suggests that taxation is a U-shaped function of the strength of the State since a weak State will anticipate his collapse and then try to appropriate the more revenues that possible before he were overthrown. As the model of incomplete property rights is static and as the event of an overthrow is not formalized, it predicts that weak States are more redistributive. [16]

The notion of strength of the State must be clarified as the definition varies between the works of Acemoglu, Kai and Skaperdas and this paper. According

[16] This discrepancy could be the base of an empirical test of restriction about the influence of the temporal horizon of the ruler.

to the former, the strength may be economic: the ability to raise taxes, or political: the degree of exposure to a revolution. Kai and Skaperdas consider the formation of the State itself and then define the strength of the State by the properties of appropriation function. The State monopolizes the legitimate violence and must be therefore the more efficient player in the use of forces [17]. These two elements are present in the model: β reflects the Acemoglu's notion of economic and political strength of the State whereas properties of the CSF deal with the crux of economic theory of the State. However, the State and the non ruling group are identically skilled for fighting so the strength of the State cannot be well-defined in this sense. In despite of this limitation, it can be argued that most of legal armies in the developing world have no access to very superior technologies than rebels. Those countries can be characterized by a high level of fixed effect "d" in their CSF which reduces the impact of technologies heterogeneity among the contestants.

If we define the civil war by the specialization of the non ruling group in the contest, this socially and human dramatic outcome occurs when the State is strong and autocratic and when the minority is weakly efficient in production and appropriation. It is commonly admitted that the civil war is the result of the abnormally high level of efficiency of the rebels in the use of arms. In contrast, the conclusion of this paper is that civil war is driven by the high discrimination of a weak group whose last resort is the contest. Moreover, the likelihood of civil war is reduced and not enhanced by the presence of valuable resources given that those resources are contestable.

[17] It may be rational for all players in the "state of nature" that the player having the more skills in the activity of protection (appropriation) will monopolize its use. This player becomes progressively a Lord then a State.

6 Conclusion

The model draws its originality from the generalization of rent-seeking to production interactions. The aim of the approach was to give a rigorous formalization of the "greed-driven" civil conflict, a contemporaneous reality, with an emphase on the interactions between cooperation and conflict. I borrowed some features of the economics models of conflict, especially the joint production function. The critical assumption of the paper is that a State may have only a partial control of its jurisdiction exposing the ruler to a threat of rebellion. This paper provides the proof of the existence and the uniqueness of a Nash equilibrium in this game. The interior equilibrium, conditional upon an incomplete enforcement of property rights, can be conflicting or peaceful, symmetric or asymmetric. This double classification allows to reduce a large number of civil conflict cases in the equilibrium. Generalized conflict are due to a collapse of the State while one-sided rebellion is the result of the coexistence between a strong autocratic regime and a weak minority. The negative role of natural resources and economic discrimination are both confirmed. The analysis of taxation choice by the ruler stresses the necessity to struggle for democratization since strong autocratic regimes are both unfair and prone to civil conflict. In addition, natural resources have a negative impact if they are not controlled by the State, increasing the threat of rebellion. As a result, the ruler is expected to adopt a quite benevolent attitude toward the minority, preventing outbreaks of civil wars. An extension of the model to dynamic framework with an endogeneization of the strength of the State, maybe dependent to the past level of contest, could be the base of a future work.

A Appendix

Proof. Lemma 1

Debreu (1952)'s Theorem states that if each payoff is quasi-concave in players's own efforts on the interval set of the actions and if the set of feasible actions for each player is convex and compact, there exists at least a pure Nash Equilibrium. As the strategies lie in the interval [0,1], this one is convex and compact. In addition, as the payoffs are concave, they are obviously quasi-concave too. ■

Proof. Theorem 1

Guaranteeing uniqueness and global stability of Nash equilibrium is not straightforward and several strategies are available to show it. I use Rosen theorem (65) who showed that under strict diagonal concavity of payoff, Nash equilibrium is unique and convergent. Moulin (86) presents a computational method to check diagonal concavity. In the two person case, it is sufficient to show that the following inequality holds for all $x \in X$ and for payoff strictly concave:

$$\left|\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2} + \frac{\partial^2 \pi^2}{\partial x_1 \partial x_2}\right| \leq 2 \left|\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2} \bullet \frac{\partial^2 \pi^2}{\partial x_1 \partial x_2}\right|^{1/2} \tag{A1}$$

Hence, after developing and regrouping terms of RHS, one can rewrite (A1) as:

$$\begin{aligned} |(1-\beta) f_{12}| \leq 2 \,|&-p_{11} p_{22} (\beta T)^2 + \\ &\beta T (1-\beta)((1-\alpha) p_{11} f_{22} - \alpha p_{22} f_{11}) + \\ &(1-\beta)^2 \alpha (1-\alpha) f_{11} f_{22}|^{1/2} \end{aligned} \tag{A2}$$

Note that all the terms of the RHS of (A2) are of same sign so that if we show that LHS is inferior or equal to any of the components of RHS, the uniqueness is guaranteed.

For $x = 0$, we know that $f_{ii} \to 0$ as well as f_{12}, condition (A2) is then satisfied.

For $x = 1$, $f_{12} \to 0$ so (A2) holds too.

As f is linearly homogeneous, its partial derivatives are homogeneous of degree 0. Then, we can write $f_{12}^2 = f_{11}f_{22}$. Substituting and rearranging gives:

$$\left|(1-\beta)^2 f_{12}^2\right| \leq$$
$$4\left|-p_{11}p_{22}(\beta T)^2 + \beta T(1-\beta)(1-\alpha)p_{11}\frac{f_{12}^2}{f_{11}}\right.$$
$$\left. - \beta T(1-\beta)\alpha p_{22}\frac{f_{12}^2}{f_{22}} + (1-\beta)^2\alpha(1-\alpha)f_{12}^2\right| \quad \text{(A3)}$$

At this stage, we can easily check that for $\beta = 1$, $\alpha = 1/2$ and $f_{12} = 0$, the previous inequality holds.

The further step is to isolate the second term of RHS and to move its last term in LHS. Then, we obtain the following condition:

$$\left|(1-\beta)^2 f_{12}^2(1-\alpha(1-\alpha))\right| \leq 4\left|\beta T(1-\beta)(1-\alpha)p_{11}\frac{f_{12}^2}{f_{11}}\right| \quad \text{(A4)}$$

which one can rewrite as:

$$|(1-\beta)(1-\alpha(1-\alpha))| \leq 4\left|\beta T(1-\alpha)\frac{p_{11}}{f_{11}}\right| \quad \text{(A5)}$$

As the LHS cannot exceeds one, we obtain the following condition:

$$\left|\beta T(1-\alpha)\tfrac{p_{11}}{f_{11}}\right| \geq \tfrac{1}{4}$$

Following similar steps with the third term of (A3), gives:

$$\left|-\beta T\alpha\tfrac{p_{22}}{f_{22}}\right| \geq \tfrac{1}{4}$$

At this stage, it is straightforward to modify the two previous inequalities as those in theorem 1 given that those conditions apply for $x \in]0,1[$

QED ■

Proof. Proposition 3

As the determinant is positive, the sign of the comparative statics depends only upon the sign the numerator of $\partial x_i/\partial z$

Numerator of $\partial x_1^*/\partial T = N_T^1$

$N_T^1 = \beta[-((1-\beta)(1-\alpha)f_{22} - \beta T p_{22})p_1 + p_2((-1+\beta)\alpha f_{12} - \beta T p_{12})]$

$$N_T^1 = \beta\left[-\left(\underbrace{\frac{\partial^2 \pi^2}{\partial x_2^2}}_{<0}\right)\underbrace{p_1}_{>0} - \underbrace{p_2}_{<0}\left(\underbrace{\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2}}_{?}\right)\right]$$

If $\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2} \geq 0$, $N_T^1 > 0$

If $\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2} < 0$, $N_T^1 \geq 0$ if $-\frac{p_1}{p_2} \geq \frac{\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2}}{\frac{\partial^2 \pi^2}{\partial x_2^{\acute{e}}}}$

Numerator of $\partial x_2^*/\partial T$

$N_T^2 = \beta[p_1((1-\beta)(1-\alpha)f_{12} - \beta T p_{12}) + p_2((1-\beta)\alpha f_{11} + \beta T p_{11})]$

$$N_T^2 = \beta\left[\underbrace{p_1}_{>0}\left(\underbrace{\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2}}_{?}\right) + \underbrace{p_2}_{<0}\left(\underbrace{\frac{\partial^2 \pi^1}{\partial x_1^2}}_{<0}\right)\right]$$

If $\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2} \geq 0$, $N_T^2 > 0$

If $\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2} < 0$, $N_T^2 \geq 0$ if $-\frac{p_2}{p_1} \geq \frac{\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2}}{\frac{\partial^2 \pi^1}{\partial x_1^{\acute{e}}}}$

Numerator of $\partial x_1^*/\partial \beta$

$N_\beta^1 = -[((1-\beta)(1-\alpha)f_{22} - \beta T p_{22})(-\alpha f_1 + T p_1) + ((-1+\alpha)f_2 - T p_2)((-1+\beta)\alpha f_{12} - \beta T p_{12})]$

$$N_\beta^1 = -\left[\left(\underbrace{\frac{\partial^2 \pi^2}{\partial x_2^2}}_{<0}\right)\underbrace{(-\alpha f_1 + T p_1)}_{>0} - \underbrace{(-1+\alpha)f_2 - T p_2)}_{>0}\left(\underbrace{\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2}}_{?}\right)\right]$$

If $\frac{\partial^2 \pi^1}{\partial x_1 \partial x_2} \geq 0$, then $N_\beta^1 > 0$

If $\frac{\partial^2\pi^1}{\partial x_1 \partial x_2} < 0$, then $N_\beta^1 \geq 0$ if $\frac{(-\alpha f_1 + Tp_1)}{(-1+\alpha)f_2 - Tp_2)} \geq \frac{\frac{\partial^2\pi^1}{\partial x_1 \partial x_2}}{\frac{\partial^2\pi^2}{\partial x_2^2}}$

Numerator of $\partial x_2^*/\partial\beta$

$N_\beta^2 = -[(-\alpha f_1 + Tp_1)(-(1-\beta)(1-\alpha)f_{12} + \beta Tp_{12}) + ((-1+\alpha)f_2 - Tp_2)((1-\beta)\alpha f_{11} + \beta Tp_{11})]$

$$N_\beta^2 = -\left[-\underbrace{(-\alpha f_1 + Tp_1)}_{>0}\left(\underbrace{\frac{\partial^2\pi^2}{\partial x_1 \partial x_2}}_{?}\right) + (\underbrace{((-1+\alpha)f_2 - Tp_2)}_{>0}\left(\underbrace{\frac{\partial^2\pi^1}{\partial x_1^2}}_{<0}\right)\right]$$

If $\frac{\partial^2\pi^2}{\partial x_1 \partial x_2} \geq 0$, then $N_\beta^2 > 0$

If $\frac{\partial^2\pi^2}{\partial x_1 \partial x_2} < 0$, then $N_\beta^2 \geq 0$ if $\frac{(-1+\alpha)f_2 - Tp_2)}{(-\alpha f_1 + Tp_1)} \geq \frac{\frac{\partial^2\pi^2}{\partial x_1 \partial x_2}}{\frac{\partial^2\pi^1}{\partial x_1^2}}$

Numerator of $\partial x_1^*/\partial\alpha$

$N_\alpha^1 = (1-\beta)[-((1-\beta)(1-\alpha)f_{22} - \beta Tp_{22})f_1 + f_2((-1+\beta)\alpha f_{12} - \beta Tp_{12})]$

$$N_\alpha^1 = (1-\beta)\left[-\left(\underbrace{\frac{\partial^2\pi^2}{\partial x_2^2}}_{<0}\right)\underbrace{f_1}_{<0} - \underbrace{f_2}_{<0}\left(\underbrace{\frac{\partial^2\pi^1}{\partial x_1 \partial x_2}}_{?}\right)\right]$$

If $\frac{\partial^2\pi^1}{\partial x_1 \partial x_2} \leq 0$, then $N_\alpha^1 < 0$

If $\frac{\partial^2\pi^1}{\partial x_1 \partial x_2} > 0$, then $N_\alpha^1 \leq 0$ if $\frac{f_1}{f_2} \geq -\frac{\frac{\partial^2\pi^1}{\partial x_1 \partial x_2}}{\frac{\partial^2\pi^2}{\partial x_2^2}}$

Numerator of $\partial x_2^*/\partial\alpha$

$N_\alpha^2 = (1-\beta)[f_1((1-\beta)(1-\alpha)f_{12} - \beta Tp_{12}) + f_2((1-\beta)\alpha f_{11} + \beta Tp_{11})]$

$$N_\alpha^2 = (1-\beta)\left[\underbrace{f_1}_{<0}\left(\underbrace{\frac{\partial^2\pi^2}{\partial x_1 \partial x_2}}_{?}\right) + \underbrace{f_2}_{<0}\left(\underbrace{\frac{\partial^2\pi^1}{\partial x_1^2}}_{<0}\right)\right]$$

If $\frac{\partial^2\pi^2}{\partial x_1 \partial x_2} \leq 0$, then $N_\alpha^2 > 0$

If $\frac{\partial^2\pi^2}{\partial x_1 \partial x_2} < 0$, then $N_\alpha^2 \geq 0$ if $\frac{f_2}{f_1} \geq \frac{\frac{\partial^2\pi^2}{\partial x_1\partial x_2}}{\frac{\partial^2\pi^1}{\partial x_1^2}}$ ■

Proof. Lemma 2

Recall that at symmetric equilibrium, condition iii of proposition 3 is fulfilled. Then the problem arises if $\frac{\partial \pi_1^2}{\partial x_1 \partial x_2} > 0$, i.e. if $x_1 > x_2$. As players are symmetric, this implies that $\alpha < 1/2$. Rewrite condition iii as:

$$-\left(\frac{\partial^2\pi^2}{\partial x_2^2}\right) f_1 - f_2 \left(\frac{\partial^2\pi^2}{\partial x_1 \partial x_2}\right) \leq 0 \tag{A6}$$

If the total differentiation of (A6) in respect to α is positive, it proves that when α is increasing, condition (A6) remains true and that $\frac{dx_1}{d\alpha}$ is always negative. The derivation of (A6) leads to:

$$\begin{aligned}
\gamma = \underbrace{(1-\beta)}_{>0} \Bigg(& \underbrace{\left(\frac{dx_2}{d\alpha} f_{22} + \frac{dx_1}{d\alpha} f_{12}\right)}_{<0} \underbrace{(-(1-\beta)\alpha f_{12} - \beta T p_{12})}_{<0} \\
& + \underbrace{f_1}_{<0} \Bigg[\underbrace{(1-\beta) f_{22}}_{<0} + \underbrace{\frac{dx_2}{d\alpha}(-(1-\beta)(1-\alpha) f_{222} + \beta T p_{222})}_{<0} \\
& \quad + \underbrace{(-(1-\beta)(1-\alpha) f_{122} + \beta T p_{122})}_{>0} \underbrace{\frac{dx_1}{d\alpha}}_{<0} \Bigg] \\
& + \underbrace{(-(1-\beta)(1-\alpha) f_{22} + \beta T p_{22}) \left(\frac{dx_2}{d\alpha} f_{12} + \frac{dx_1}{d\alpha} f_{11}\right)}_{>0} \\
& + \underbrace{f_2}_{<0} \Bigg[\underbrace{-(1-\beta) f_{12}}_{<0} \underbrace{-(1-\beta)\alpha}_{<0} \left(\underbrace{\frac{dx_2}{d\alpha} f_{122}}_{<0} + \underbrace{\frac{dx_1}{d\alpha} f_{112}}_{>0} \right) \\
& \quad - \beta T \left(\underbrace{\frac{dx_2}{d\alpha} p_{122}}_{>0} + \underbrace{\frac{dx_1}{d\alpha} p_{112}}_{?} \right) \Bigg] \Bigg)
\end{aligned} \tag{A7}$$

Combining assumptions 1-2 and 5, we obtain the signs appearing under elements of (A7). Only two of them remains ambiguous and it is straightforward to check that sufficient conditions for (A7) be positive are:

C1 $(\frac{dx_2}{d\alpha}f_{122} + \frac{dx_1}{d\alpha}f_{112}) \geq 0$

C2 $(\frac{dx_2}{d\alpha}p_{122} + \frac{dx_1}{d\alpha}p_{112}) \geq 0$

For $\alpha = 1/2$, C1 and C2 equal zero and (A6) is verified. When α decreases f_{112} increases while f_{122} lowers since player 1 reduces his productive effort. In addition, for the same movement of α, p_{122} raises whereas p_{112} decreases. Moreover, if $x_1 \geq 2x_2 + 2d$, p_{112} becomes negative. Then, if (A6) is true for $\alpha = 1/2$, (A6) is true also for $\alpha < 1/2$. Following similar steps will give symmetric results for $\frac{dx_2}{d\alpha} > 0$ when $\alpha > 1/2$

QED ■

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