

# Moral Hazard and Marshallian Inefficiency: Evidence from Tunisia\*

Jean-Louis ARCAND  
CERDI-CNRS  
Université d'Auvergne, and EUDN

Chunrong AI  
Department of Economics  
University of Florida

François ETHIER  
Caisses Populaires Desjardins

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## Abstract

We formalize the link between optimal cost-sharing contracts and the production technology in the presence of moral hazard by appealing to several well-known results from duality theory. Building on intuitions from the interlinkage literature, we show that optimal contractual structure is determined by the (i) substitution possibilities that exist between different observable factor inputs, as well as (ii) between these inputs and unobservable effort. We endogenize contractual choice using landlord characteristics as instruments, exploiting the fact that, in our dataset, landlords interact with several tenants and *vice versa*. The approach is applied to an unbalanced plot-level panel of cost sharing contracts in a Tunisian village, using a translog representation of the restricted profit function. Contractual terms are found to be a significant determinant of input use and therefore lead to Marshallian inefficiency, while the optimality of the underlying contractual structure is rejected.

*Keywords:* moral hazard, profit functions, sharecropping, Marshallian inefficiency

*JEL:* O12, O13, D21, D82, Q12, C33

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...when the cultivator has to give to the landlord half of the returns to each dose of capital and labour that he applies to the land, it will not be to his interest to apply any doses the total return to which is less than twice enough to reward him. If, then, he is free to cultivate as he chooses, he will cultivate far less intensively... so that his landlord will get a smaller share even of those returns than he would have on the plan of a fixed payment.... The position of a peasant proprietor has great attractions. He is free to do what he likes, he is not worried by the interference of a landlord, and the anxiety lest another reap the fruits of his work and self-denial... He is scarcely ever idle, and seldom regards his work as mere drudgery; it is all for the land that he loves so well.

Alfred Marshall (1920)  
*Principles of Economics*, 8th edn, pp. 535-7

## 1 Introduction

### 1.1 Marshallian inefficiency

Testing for the Marshallian inefficiency of agricultural contracts in less developed countries has become something of a cottage industry (see., e.g., Otsuka and Hayami (1988) for one among several excellent surveys). Amid the mass of empirical contributions, two papers stand out because of the innovative methodology they proposed: Bell (1977) and especially Shaban (1987). In these papers, the fundamental problem of assessing the productivity differential that may exist between plots under sharecropping and plots under owner-operatorship while maintaining the *ceteris paribus* assumption is addressed by considering households that farm more than one plot, and in particular households that are simultaneously owner-operators and sharecroppers. The use of household-specific fixed effects then allows one to compare the productivity of the two classes of plots while at least maintaining constant the identity of the household engaging in the farming activity. While this approach, coupled with Shaban's focus on cost-sharing contracts—which yield a much greater degree of contractual heterogeneity than the simple owner-operator / sharecropped dichotomy—has shaped our understanding of the incentive effects of tenancy contracts, several issues remain open.

### 1.2 Contracts, producers theory and interlinkage

First, contractual structure still remains largely exogenous in this literature, despite the fact that an impressive corpus of theory (and empirical results from other areas of economics) suggests otherwise. Second, despite the fact that factor demand equations are usually being estimated, no thought appears to be given to the underlying theoretical hypotheses: these factor demand equations presumably stem from some well-posed optimization problem, which may impose non-trivial restrictions on the form that the estimated relationship takes. Third, despite the seminal contributions by Braverman and Stiglitz (1982) and Braverman and Stiglitz (1986) regarding the theoretical interpretation of cost-sharing contracts in terms of interlinkage, no attempt has been made to relate optimal contractual structure to the production technology.<sup>1</sup>

The purpose of this paper, at least in the context of the debate surrounding the (in)efficiency of sharecropping contracts, is to show: (i) that the endogeneity of contractual structure does matter;

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<sup>1</sup>See also Bardhan and Singh (1987).

(ii) that factor demand equations cannot be blithely formulated in open violation of the most elementary restrictions suggested by producers theory, and (iii) that it is possible to construct a direct structural test of the optimality of the observed contracts when one couples a simple principal-agent model of moral hazard to the standard results of the duality theory of the firm.

### 1.3 Structure of the paper

The structure of this paper is as follows. In part 2, we develop a simple model of tenancy under risk-neutrality that takes the form of cost-sharing contracts and show that the usual tests of Marshallian inefficiency (such as Shaban (1987)) suffer from (i) endogeneity problems, (ii) several identifiable forms of specification error due to their neglect of the elementary restrictions that stem from the theory of the firm. We then provide a characterization of optimal contractual structure under moral hazard and show how this is intimately linked to the properties of the production technology. These restrictions are testable, given the appropriate data. We then show that it is straightforward to extend our model to risk-averse tenants (thereby accounting for the existence of sharecropping contracts when fixed rental is a viable alternative) and landlord-determined crop choice, and that the salient features of our characterization of optimal contractual structure are preserved.

In part 3, we begin by applying standard instrumental variables techniques to "Shaban" type regressions using data from a Tunisian village. We show that it is possible to endogenize contractual choice in a plot level dataset in which tenants interact with *several* landlords by using landlord characteristics, expressed in terms of deviations with respect to operator household-specific individual means (this controls for operator household unobserved heterogeneity), as instrumental variables. A battery of tests are implemented, including the recent Hahn and Hausman (2002a) test for the validity of instrumental variables. Our results suggest that output per hectare is the same on sharecropped plots than on those under owner operators or fixed rental contracts, *ceteris paribus*. These results are, however, tentative, in that they fail to impose the restrictions from the theory of the firm developed in part 2.

We then reconsider the problem using the translog parameterization of the restricted profit function, which we adapt so that it correctly accounts for unobservable effort. A first pass, in which we estimate the full system constituted by the restricted profit function and the associated factor share equations by the Seemingly Unrelated Regression (SUR) technique, confirms that there is indeed enough variability in the cost-shares for one to be able to precisely identify a significant number of own and cross-price effects. We then endogenize contractual structure, again using landlord characteristics as instrumental variables, and reject the null of no effect of the contractual variables for all ten factor inputs, as well as for output. Finally, we explicitly test the restrictions developed earlier that characterize an optimal cost-sharing contract under moral hazard, and reject.

This suggests, while the terms of the contract affect the allocation of resources and thereby do yield Marshallian inefficiency, that other mechanisms apart from the standard Principal-Agent model may be driving the particular form taken by cost-sharing contracts in the village under scrutiny. Part 4 concludes.

## 2 Formalization

Consider a simple sharecropping contract in which a landlord rents a plot of land of fixed size  $T$  in exchange for a share  $1 - \alpha$  of output. The landlord also pays a share  $1 - \beta_i$  of the cost of each factor input, indexed by  $i = 1, \dots, I$ . The landlord will be assumed to be able to set the  $I+1$  vector  $(\alpha, \beta)$ . Let the production technology be given by the additively separable form  $\theta + q$ , where  $q$  is output, and  $\theta$  is a random variable assumed to be distributed according to the probability density function (*pdf*)  $g(\theta)$ ,  $\theta \in [\underline{\theta}, \bar{\theta}]$ , with  $E[\theta] = 0$  and  $var[\theta] = \sigma^2$ .

For simplicity, and in order to focus on the pure incentive effects of contractual structure, we assume for the time being that both parties to the contract are risk-neutral, and we rule out fixed rental contracts.<sup>2</sup> Risk-averse tenants and the choice between sharecropping and fixed rental will be considered below. The landlord's (superscript  $L$ ) objective function is then given by:

$$\begin{aligned} \Pi^L &= \int_{\underline{\theta}}^{\bar{\theta}} \left[ (1 - \alpha)(\theta + q) - \sum_{i=1}^{i=I} (1 - \beta_i)X_i \right] g(\theta) d\theta \\ &= (1 - \alpha)F(T, X, e) - \sum_{i=1}^{i=I} (1 - \beta_i)X_i, \end{aligned} \quad (1)$$

where, for clarity of exposition, all prices are normalized to one,  $q = F(T, X, e)$  represents the non-stochastic portion of the production technology,  $X_i$  is the quantity of factor input  $i$  ( $X = (X_1, X_2, \dots, X_i, \dots, X_I)$ ), and  $e$  is the level of effort furnished by the tenant. The tenant's (superscript  $T$ ) objective function is given by:

$$\begin{aligned} \Pi^T &= \int_{\underline{\theta}}^{\bar{\theta}} \left[ \alpha(\theta + q) - \sum_{i=1}^{i=I} \beta_i X_i - \omega e \right] g(\theta) d\theta \\ &= \alpha F(T, X, e) - \sum_{i=1}^{i=I} \beta_i X_i - \omega e, \end{aligned} \quad (2)$$

where  $\omega e$  is the disutility of effort, expressed in monetary terms.<sup>3</sup> Note, whether we are considering the first best optimum or the situation under moral hazard, that we will assume that the landlord is fully aware of the preferences of the tenant including, most importantly, the value of  $\omega$ . The tenant will accept a contract offered by the landlord as long as his participation constraint (PC) is satisfied. We write this as:

$$\Pi^T \geq \bar{\Pi}^T, \quad (3)$$

where  $\bar{\Pi}^T$  represents the reservation level of the tenant's expected net payoffs measured in mon-

<sup>2</sup>This is because, in the absence of a risk-averse tenant or market imperfections such as credit constraints, the landlord could achieve the first-best optimum with a risk-neutral tenant simply by offering a fixed rental contract that would guarantee the tenant his reservation level of welfare. In the notation used here, this would correspond to setting  $\alpha = \beta_1 = \dots = \beta_i = \dots = \beta_I = 1$ , and adding a fixed transfer to the tenant's objective function. Risk-aversion will be considered in section 2.5. We will examine the choice between fixed rental and sharecropping in the context of the present model in section 2.6, once risk-aversion has been introduced.

<sup>3</sup>The disutility of effort is linear in effort in order to facilitate the application, in what follows, of standard results from duality theory, since the marginal disutility of effort,  $\omega$  can simply be interpreted as another "price". An alternative specification would involve effort appearing in multiplicative form in the production function (i.e.  $F(T, X, e) = eF(T, X)$ ), while its disutility would take the form  $\omega(e)$ ,  $\omega'(e) > 0$ ,  $\omega''(e) > 0$ . In mathematical terms, either specification would be valid, and simply corresponds to a particular normalization of the variable "effort". In terms of analytical tractability, however, the chosen specification is much more convenient.

etary terms, which will be a function of a vector of the tenant's characteristics (such as outside opportunities), as well as the landlord's (for example, resident landlords may be able to drive a harder bargain than those of the absentee ilk).

## 2.1 The first-best optimum

When the tenant's actions are observable to the landlord, the solution to the landlord's optimization problem is given by:

$$(\alpha^{**}, \beta^{**}, X^{**}, e^{**}) = \arg \max_{\{\alpha, \beta, X, e\}} \Pi^L \text{ s. t. } \Pi^T \geq \bar{\Pi}^T \quad (4)$$

where  $\beta = (\beta_1, \beta_2, \dots, \beta_i, \dots, \beta_I)$ . Substituting from the participation constraint, this problem may be expressed in unconstrained form as:

$$\max_{\{X, e\}} F(T, X, e) - \sum_{i=1}^{i=I} X_i - \omega e - \bar{\Pi}^T. \quad (5)$$

The vector  $(\alpha^{**}, \beta^{**}, X^{**}, e^{**})$  is then implicitly defined by the  $I + 1$  first-order conditions (FOCs):

$$F_{X_i}(T, X^{**}, e^{**}) - 1 = 0, \quad i = 1, \dots, I; \quad (6)$$

$$F_e(T, X^{**}, e^{**}) - \omega = 0; \quad (7)$$

(where the subscripts on  $F(\cdot)$  denote partial derivatives) plus the participation constraint:

$$\Pi^{T^{**}} = \alpha^{**} q^{**} - \sum_{i=1}^{i=I} \beta_i^{**} X_i^{**} - \omega e^{**} = \bar{\Pi}^T, \quad (8)$$

where  $q^{**} = F(T, X^{**}, e^{**})$  is the tenant's supply function. As should be obvious from conditions (6) and (7), the cost share terms (i.e., the elements of vector  $\beta$ ), as well as the output-share  $\alpha$ , play no role in terms of the incentives that determine factor inputs and effort, and serve merely to ensure that the participation constraint is satisfied, through condition (8). One may therefore write:

$$X^{**} = X^{**}(T, \omega, \bar{\Pi}^T), \quad (9)$$

$$e^{**} = e^{**}(T, \omega, \bar{\Pi}^T), \quad (10)$$

$$q^{**} = q^{**}(T, \omega, \bar{\Pi}^T). \quad (11)$$

Consider the expected net payoff of the tenant expressed in monetary terms, at the optimum, as given by the left-hand-side (LHS) of (8). This can be decomposed into two parts:

$$\Pi^{T^{**}} = \tilde{\Pi}^{T^{**}} - \omega e^{**}, \quad (12)$$

where  $\tilde{\Pi}^{T^{**}}$  is *not* the usual profit function familiar from received microeconomic theory, whereas  $\Pi^{T^{**}}$  *is*. In order to avoid confusion in what follows, we will denote  $\tilde{\Pi}^{T^{**}}$  by the term "observable profit function", in contrast to  $\Pi^{T^{**}}$  which includes the *non-observable* component of profits given by the opportunity cost of effort. This distinction will be important in what follows because

the landlord and, more importantly for our purposes, the econometrician, does not observe  $\Pi^{T**}$  but is able to observe  $\tilde{\Pi}^{T**}$ , *ex post*. Then straightforward differentiation, since  $X^{**}$  and  $e^{**}$  are independent of  $(\alpha, \beta)$ , yields:

$$\frac{d\Pi^{T**}}{d\alpha} = \frac{d\tilde{\Pi}^{T**}}{d\alpha} = q^{**}, \quad \frac{d\Pi^{T**}}{d\beta_i} = \frac{d\tilde{\Pi}^{T**}}{d\beta_i} = -X_i^{**}, \quad i = 1, \dots, I. \quad (13)$$

Note that the Envelope Theorem does *not* need to be invoked here in carrying out these comparative statics given the independence of factor demands with respect to  $(\alpha, \beta)$ .

## 2.2 Moral hazard with delegation

When the landlord cannot observe the tenant's actions, he faces additional constraints stemming from the tenant's optimal choice of factor inputs and effort.<sup>4</sup> In the tradition of Holmström (1979) and Harris and Raviv (1979), write the solution to the landlord's optimization problem in this case as:

$$\begin{aligned} (\alpha^*, \beta^*, X^*, e^*) &= \arg \max_{\{\alpha, \beta, X, e\}} \Pi^L & (14) \\ \text{s.t.} &\begin{cases} \Pi^T \geq \bar{\Pi}^T \quad (PC) \\ (X^*, e^*) = \arg \max_{\{X, e\}} \Pi^T \quad (IC) \end{cases} \end{aligned}$$

Consider the incentive compatibility (*IC*) constraint:

$$(X^*, e^*) = \arg \max_{\{X, e\}} \Pi^T = \arg \max_{\{X, e\}} \alpha F(T, X, e) - \sum_{i=1}^{i=I} \beta_i X_i - \omega e. \quad (15)$$

This is characterized by the following  $I + 1$  FOCs:<sup>5</sup>

$$\alpha F_{X_i}(T, X^*, e^*) - \beta_i = 0, \quad i = 1, \dots, I; \quad (16)$$

$$\alpha F_e(T, X^*, e^*) - \omega = 0. \quad (17)$$

In contrast to the case without moral hazard, the solution to equations (16) and (17), along with the participation constraint, yields:

$$X^* = X^*(T, \omega, \Pi^T, \alpha, \beta), \quad (18)$$

$$e^* = e^*(T, \omega, \Pi^T, \alpha, \beta), \quad (19)$$

$$q^* = q^*(T, \omega, \Pi^T, \alpha, \beta), \quad (20)$$

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<sup>4</sup>In what follows, we assume that the tenant chooses  $e$  and  $X$ . Formally, in terms of the well-known delegation argument suggested by Braverman and Stiglitz (1986), this will obtain only when the tenant possesses an informational advantage over the landlord thus making delegation by the latter to the former optimal. The results that follow do not differ appreciably in qualitative terms if the landlord is assumed to be able to choose  $X$ . The detailed characterization of the optimal contract furnished in PROPOSITION 1 is, however, slightly modified. Note that we abstract from the important insight of Bardhan and Singh (1987) concerning the absence of incentive effects of cost sharing *at the margin*, when the landlord chooses  $X$ .

<sup>5</sup>For simplicity, we shall assume that the first-order approach is valid; see Jewitt (1988).

where  $q^* = F(T, X^*(T, \omega, \Pi^T, \alpha, \beta), e^*(T, \omega, \Pi^T, \alpha, \beta))$ . The optimal levels of physical inputs and effort chosen by the tenant are now functions of the terms of the cost-sharing contract, as is output. The difference between equations (9) and (18) constitutes the basis for the classic tests of Marshallian inefficiency carried out by Bell (1977) and Shaban (1987). In the absence of moral hazard, optimal input use (as well as output) should be *independent* of the terms of the contract  $(\alpha, \beta)$ .

### 2.3 Why the standard approach is wrong

A first important weakness of such tests, as should be obvious from the optimization problem posed in (14), is that estimating factor demand equations of the form  $X_i^* = X_i^*(T, \omega, \Pi^T, \alpha, \beta)$  leads one straight into a potentially serious endogeneity problem, because  $\alpha$  and  $\beta$  are not randomly chosen by nature and corresponds to part of the solution to the problem. In other words: carrying the reasoning begun with the landlord's optimization problem through to its logical conclusion, it should be obvious that  $\alpha^* = \alpha^*(T, \omega, \bar{\Pi}^T)$  and  $\beta^* = \beta^*(T, \omega, \bar{\Pi}^T)$ . Substituting these last expressions into (18) thus leads to the full solution in terms of exogenous variables, which takes the form:

$$X^* = X^*(T, \omega, \bar{\Pi}^T). \quad (21)$$

In analytical and, most importantly, in econometric terms, equations (9) and (21) are indistinguishable. The reduced form representations of the factor demand equations do **not** allow one to test whether moral hazard is present. The upshot should be clear: a factor demand equation which takes the form  $X_i^* = X_i^*(T, \omega, \Pi^T, \alpha, \beta)$  should be estimated as a structural equation in which appropriate instruments must be found for the terms of the contract.<sup>6</sup>

A second weakness of existing tests for Marshallian inefficiency stems from their neglect of the elementary restrictions on functional form that stem from received producers theory. In concrete terms, regressing  $X_i^*$  on the vector  $(\alpha, \beta)$  (as well as plot-level controls and household-specific effects, if possible) is neither here nor there in the sense that such a linear specification does not correspond *under any circumstances* to a well-behaved factor demand equation. In particular, given that one should interpret  $\beta_i$  as the "effective" factor price, faced by the tenant, for input  $i$ , and  $\alpha$  as the "effective" output price, the equation being estimated should at least satisfy the homogeneity property of a factor demand equation (see, e.g., Varian (1978)). Moreover, the factor demand equations should be estimated as a system using the SUR approach, as is done in conventional microeconomic work associated with the theory of the firm (see, e.g., Fuss and McFadden (1978)), and the standard symmetry restriction on the cross partial derivatives should also be imposed. To the best of our knowledge, these restrictions have never been imposed in the context of the Marshallian inefficiency debate.

Carrying out the standard regression of  $X_i^*$  on  $\beta$  (or  $(\beta_1/\alpha, \dots, \beta_i/\alpha, \dots, \beta_I/\alpha)$ ) and plot characteristics therefore constitutes a gross error in specification, and any conclusions one may try to draw from the estimated coefficients associated with  $\beta$  may simply be driven by what amounts to endogeneity or specification bias.

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<sup>6</sup>The same is true of the output supply equation, as given in (20). To be precise, one needs at least  $I$  instrumental variables in that the  $I$  physical factor demand equations  $X^*(T, \alpha, \beta, \omega)$ , are functions of the *ratios*  $(\frac{\beta_1}{\alpha}, \dots, \frac{\beta_i}{\alpha}, \dots, \frac{\beta_I}{\alpha})$ .

## 2.4 Contractual choice and the production technology

The main original contribution of this paper, apart from providing an empirical application in which Marshallian efficiency is tested while endogenizing contractual structure and using the restrictions that flow from the apparatus of producers theory (more on the mechanics of this in part 3 of the paper), is to formally test the optimality of contractual structure, as it is set out in a standard principal-agent framework, by focusing on the landlord's optimal response to the non-observability of tenant effort. Such optimizing behavior, if it is present, should lead to an intimate structural link between the optimal contract and the underlying production technology.

### 2.4.1 A characterization of the optimal contract

The point of departure is the constrained optimization problem faced by the landlord in (14). The following PROPOSITION, which is driven in part by the linear homogeneity property of the restricted profit function, provides a characterization of the optimal cost-sharing contract in terms that should be easy to interpret from the standpoint of received duality theory.

**Proposition 1** (i) *The optimal cost-shares, denoted by the  $1 \times I$  vector  $\beta^*$ , are characterized by the following restrictions:*

$$\frac{dq^*}{d\beta_i} - \sum_{j=1}^{j=I} \frac{dX_j^*}{d\beta_i} - \omega \frac{de^*}{d\beta_i} = 0, i = 1, \dots, I,$$

which can be re-expressed as (ii):

$$(1 - \alpha^*) \frac{dq^*}{d\beta_i} - \sum_{j=1}^{j=I} (1 - \beta_j^*) \frac{dX_j^*}{d\beta_i} = 0, i = 1, \dots, I;$$

(iii) *the optimal output-share  $\alpha^*$  is characterized by the restriction:*

$$\frac{dq^*}{d\alpha} - \sum_{j=1}^{j=I} \frac{dX_j^*}{d\alpha} - \omega \frac{de^*}{d\alpha} = 0,$$

which can be re-expressed as (iv):

$$(1 - \alpha^*) \frac{dq^*}{d\alpha} - \sum_{j=1}^{j=I} (1 - \beta_j^*) \frac{dX_j^*}{d\alpha} = 0;$$

(v) *if the landlord chooses plot size, its optimal value is characterized by:*

$$\frac{dq^*}{dT} - \sum_{j=1}^{j=I} \frac{dX_j^*}{dT} - \omega \frac{de^*}{dT} = 0.$$

PROOF: *see Appendix.*

It should be obvious that other motivations, aside from a response by the principal to the agent's opportunistic behavior with respect to effort, may also be driving contractual choice. As such, we shall go to considerable pains in the empirical portion of the paper to provide instrumental variables estimates that do not put all of the onus of the proof on the restrictions provided by PROPOSITION 1. On the other hand, the explicit characterization of the optimal contract given



by PROPOSITION 1 potentially provides one with a structural test of the empirical validity of the principal-agent approach to sharecropping, conditional of course on the maintained hypothesis that the chosen parameterization of the production technology is correct.

The characterization of optimal contractual structure given by PROPOSITION 1 is easy to understand in terms of the standard duality theory of the firm (see, e.g., Diewert (1982)). First, note that the elements in the summation in parts (i) and (ii) of PROPOSITION 1 are given by expressions of the form  $\frac{dX_j^*}{d\beta_i}$  which, by Hotelling's Lemma, correspond to the elements of a portion of the Hessian matrix of second derivatives of the restricted profit function of the tenant (evaluated at  $(\alpha^*, \beta^*)$ ), with typical element:

$$\frac{dX_j^*}{d\beta_i} = -\frac{d^2\Pi^{T*}}{d\beta_i d\beta_j} = \frac{dX_i^*}{d\beta_j}. \quad (22)$$

Second, the derivatives of the supply function ( $q^*$ ) with respect to the cost-shares, again in parts (i) and (ii) of PROPOSITION 1, correspond to elements of the Hessian matrix of the form:

$$-\frac{dq^*}{d\beta_i} = -\frac{d^2\Pi^{T*}}{d\alpha d\beta_i} = \frac{dX_i^*}{d\alpha}, \quad (23)$$

while, in part (i):<sup>7</sup>

$$\frac{de^*}{d\beta_i} = -\frac{d^2\Pi^{T*}}{d\omega d\beta_i} = \frac{dX_i^*}{d\omega}. \quad (24)$$

Parts (ii) and (iv) of PROPOSITION 1, which stem from Euler's Theorem as applied to the restricted profit function of the tenant, are particularly clear (and initially somewhat counterintuitive) in terms of the incentives put in place by the landlord: at the optimum, the landlord sets  $(\alpha^*, \beta^*)$  so that the *marginal* impact of each  $\beta_i$  on the value of the share of output he receives is just offset by the sum of the marginal impacts of  $\beta_i$  on the share of the cost of each input he bears, while *seemingly* ignoring the marginal impact of the contract on the opportunity cost of effort (a similar line of reasoning applies to  $\alpha$ ).

PROPOSITION 1 shows that it is possible, given an appropriate parameterization of the production technology through its dual representation in terms of the restricted profit function (and thus the corresponding factor demand –and supply– equations), to construct a full empirical characterization of optimal contractual structure. The latter will be intimately linked to the production technology itself. In particular, those elements stemming from (22) correspond to substitution possibilities between physical factor inputs, whereas those elements given by (24) correspond to substitution possibilities between physical factor inputs and effort. For those familiar with the literature on interlinkage (Braverman and Stiglitz (1982) and Braverman and Stiglitz (1986)), the intuitive appeal of this characterization should be obvious.

#### 2.4.2 On the importance of elasticities of substitution

What are the implications of PROPOSITION 1 in terms of the form taken by optimal cost-sharing contracts, and can the form of the optimal contract be related to a given characteristic of the

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<sup>7</sup>Note also, in parts (iii) and (iv) of the PROPOSITION, that:  $\frac{dX_j^*}{d\alpha} = -\frac{d^2\Pi^{T*}}{d\alpha d\beta_j} = -\frac{dq^*}{d\beta_j}$ ,  $\frac{dq^*}{d\alpha} = \frac{d^2\Pi^{T*}}{d\alpha^2}$ ,  $\frac{de^*}{d\alpha} = -\frac{d^2\Pi^{T*}}{d\alpha d\omega} = -\frac{dq^*}{d\omega}$ .

production technology?

As an illustration, consider a simple Cobb-Douglas production technology with two physical inputs plus effort:  $q = AX_1^{\delta_1} X_2^{\delta_2} e^{\delta_\omega}$ . Ignoring multiplicative constants, the restricted profit function (for the tenant) associated with this production technology is given by:

$$\Pi^{T*} = \left[ \alpha \beta_1^{-\delta_1} \beta_2^{-\delta_2} \omega^{-\delta_\omega} A \right]^{\frac{1}{1-\delta_1-\delta_2-\delta_\omega}}. \quad (25)$$

For this simple example, it is straightforward to show that the two conditions given by PROPOSITION 1 (i) that characterize the optimal cost-sharing contract reduce (dividing one condition by the other) to  $\beta_1 = \beta_2$ . The *level* of the cost-shares is then deduced from the binding participation constraint.

The Cobb-Douglas production technology thus results in optimal cost-shares that are equal across factor inputs. The same is true for a second commonly used production technology, the CES production function, which suggests that the conditions given in PROPOSITION 1 will yield heterogeneity in optimal cost-shares only in the presence of heterogeneity in elasticities of substitution.

In order to address this issue, the choice of an appropriate functional form for the profit function is crucial. We turn to this question in section 3.3.1.

## 2.5 Risk-aversion

We now relax the assumption of a risk-neutral tenant. In this case, the solution to the landlord's optimization problem in the presence of moral hazard is given by:

$$\begin{aligned} (\alpha^*, \beta^*, X^*, e^*) &= \arg \max_{\{\alpha, \beta, X, e\}} E[\Pi^L] & (26) \\ \text{s.t.} &\begin{cases} E[U(\Pi^T)] \geq \bar{\Pi}^T & (PC) \\ (X^*, e^*) = \arg \max_{\{X, e\}} E[U(\Pi^T)] & (IC) \end{cases} \end{aligned}$$

where  $U(\cdot)$  represents the tenant's increasing and concave utility function,  $\bar{\Pi}^T$  should now be interpreted as the tenant's reservation level of welfare, while the landlord is assumed to remain risk-neutral. Consider the incentive compatibility (IC) constraint more explicitly:

$$(X^*, e^*) = \arg \max_{\{X, e\}} \int_{\underline{\theta}}^{\bar{\theta}} U \left( \begin{array}{c} \alpha(\theta + F(T, X, e)) \\ - \sum_{i=1}^{i=I} \beta_i X_i - \omega e \end{array} \right) g(\theta) d\theta. \quad (27)$$

The associated FOCs are given by:

$$[\alpha F_{X_i}(T, X^*, e^*) - \beta_i] \int_{\underline{\theta}}^{\bar{\theta}} U'(\cdot) g(\theta) d\theta = 0, \quad i = 1, \dots, I, \quad (28)$$

$$[\alpha F_e(T, X^*, e^*) - \omega] \int_{\underline{\theta}}^{\bar{\theta}} U'(\cdot) g(\theta) d\theta = 0, \quad (29)$$

which are identical to (16) and (17) under the relatively mild assumption of non-satiation ( $U'(\cdot) > 0$ ).<sup>8</sup> The characterization of the optimal contract that would follow is, however, less than transparent, and is therefore less amenable to structural testing. For this reason, we assume that (i) the tenant's utility function displays constant absolute risk-aversion (CARA)  $\phi$ , and that (ii)  $g(\theta)$  corresponds to the normal density ( $\theta \sim N(0, \sigma^2)$ ).<sup>9</sup> Then the usual properties of the characteristic function, or Fourier transform, of the normal distribution, along with some straightforward algebraic manipulations, imply that the tenant's objective function can be rewritten as:<sup>10</sup>

$$-\ln(-E[U(\Pi^T)]) = \phi \left[ \alpha F(T, X, e) - \sum_{i=1}^{i=I} \beta_i X_i - \omega e \right] - \frac{1}{2} \phi^2 \alpha^2 \sigma^2, \quad (30)$$

where  $-\ln(-(\cdot))$  is a monotonically increasing transformation. The restrictiveness of these assumptions concerning tenant preferences and the form taken by the stochastic production technology may be open to debate, but it is clear that they allow us to maintain our use of the well-developed corpus of producers theory when it comes to characterizing optimal contractual structure.

Consider once more the landlord's optimization problem given in (26), in light of (30). We then have the following PROPOSITION.

**Proposition 2** *Assume that the tenant's preferences satisfy CARA and that the additive stochastic shock to output is distributed  $N(0, \sigma^2)$ . Then (i) the optimal cost-shares are characterized by the same restrictions as in PROPOSITION 1, and the optimal output-share is characterized by (ii):*

$$\frac{dq^*}{d\alpha} - \sum_{i=1}^{i=I} \frac{dX_i^*}{d\alpha} - \omega \frac{de^*}{d\alpha} - \phi \alpha^* \sigma^2 = 0.$$

or (iii):

$$(1 - \alpha^*) \frac{dq^*}{d\alpha} - \sum_{i=1}^{i=I} (1 - \beta_i^*) \frac{dX_i^*}{d\alpha} - \phi \alpha^* \sigma^2 = 0,$$

where  $\phi$  is the tenant's Arrow-Pratt coefficient of absolute risk-aversion.

PROOF: see Appendix.

The main change in the characterization of the optimal sharecropping contract brought about by the introduction of tenant risk-aversion is, unsurprisingly, that the optimal share of output  $\alpha^*$  becomes a function of  $\sigma^2$  and  $\phi$ . On the other hand, the interpretation of the optimal contract in the language of standard duality theory, as set forth after PROPOSITION 1, is preserved.

<sup>8</sup>The same is true of the first best optimum, where the introduction of risk aversion, in the presence of an additive stochastic, shock yields the same characterization of  $(X^{**}, e^{**})$  as in (6) and (7).

<sup>9</sup>Ignoring multiplicative constants, a CARA utility function corresponds to  $U(z) = -\exp\{-\phi z\}$ .

<sup>10</sup>For  $\mathbf{x} \sim N(\mu, \Sigma)$ , the characteristic function is  $\varphi_{\mathbf{x}}(\mathbf{t}) = \exp\{\iota \mathbf{t}'\mu - \frac{1}{2} \mathbf{t}'\Sigma \mathbf{t}\}$  where  $\iota = \sqrt{-1}$ . It is this functional form that then allows one to write  $\int_{-\infty}^{+\infty} -\exp\{-\phi x\} N(\mu, \sigma^2) dx = -\exp\{-\phi\mu + \frac{1}{2}\phi^2\sigma^2\}$ ; see, e.g. any standard Mathematical Statistics text, such as Roussas (1997). Note that a key assumption here is the additively separable form taken by the stochastic production technology, since a production technology of the form  $\theta F(T, X, e)$  with  $\theta \sim N(1, \sigma^2)$  would imply that:

$$-\log(-E[U(\Pi^T)]) = \phi \left[ \alpha F(T, X, e) - \sum_{i=1}^{i=I} \beta_i X_i - \omega e - R \right] - \frac{1}{2} \phi^2 \alpha^2 [F(T, X, e)]^2 \sigma^2.$$

The characterization of  $(X^*, e^*)$  would then be substantially different, and we would no longer be able to apply the results of received producers theory. Given that we have no reason to prefer the multiplicative over the additive form of the stochastic shock to output, we prefer to remain with the simpler specification.

Risk-aversion on the part of the tenant, coupled with the assumptions of PROPOSITION 2, also allow one to consider cropping as a landlord choice, in an extremely simple framework. Assume that several different crops are available, and that crop  $k$  is characterized by  $\theta_k \sim N(\mu_k, \sigma_k^2)$  where  $\mu_k$  is the mean level of the stochastic shock to output and  $\sigma_k^2$  is its variance; the distribution of  $\theta_k$  may, of course, be a function of plot characteristics such as soil type. Then the solution to the landlord's problem is given by:

$$(\alpha^*, \beta^*, \mu_k^*, \sigma_k^{2*}) = \arg \max_{\{\alpha, \beta, \mu_k, \sigma_k^2\}} (1 - \alpha)(\mu_k + q^*) - \sum_{i=1}^{i=I} (1 - \beta_i) X_i^* \quad (31)$$

$$s.t. \quad \phi \left[ \begin{array}{c} \alpha(\mu_k + q^*) \\ - \sum_{i=1}^{i=I} \beta_i X_i^* - \omega e^* \end{array} \right] - \frac{1}{2} \phi^2 \alpha^2 \sigma_k^2 = -\ln(-\bar{\Pi}^T),$$

where

$$(X^*, e^*) = \arg \max_{\{X, e\}} \phi \left[ \begin{array}{c} \alpha(\mu_k + F(T, X, e)) \\ - \sum_{i=1}^{i=I} \beta_i X_i - \omega e \end{array} \right] - \frac{1}{2} \phi^2 \alpha^2 \sigma_k^2. \quad (32)$$

The vector  $(X^*, e^*)$  therefore continues to be characterized by (16) and (17). It is then easy to verify that the optimal contract  $(\alpha^*, \beta^*)$  that emerges is a straightforward extension of the characterization provided in PROPOSITION 2, although it will now also depend on  $\mu_k^*$ , which is itself endogenously determined.

## 2.6 Sharecropping, fixed rental contracts, and the decision to rent out

Now that risk-aversion has been introduced, it is possible to formally address the issue of the choice between fixed rental and sharecropping contracts. In the case of a fixed rental contract, the tenant is residual claimant, with  $\alpha = \beta_i = 1, i = 1, \dots, I$ . The binding participation constraint implies that the optimal rental payment  $R^{**}$  is given by:

$$R^{**} = q^{**} - \sum_{i=1}^{i=I} X_i^{**} - \omega e^{**} - \frac{1}{2} \phi \sigma^2 + \frac{1}{\phi} \ln(-\bar{\Pi}^T), \quad (33)$$

where the tenant chooses the first-best values  $(X^{**}, e^{**})$ . Under sharecropping, the landlord's objective function evaluated at the optimum (using the manipulations detailed in the proof of PROPOSITION 1) is given by:

$$E[\Pi^{L*}] = q^* - \sum_{i=1}^{i=I} X_i^* - \omega e^* - \frac{1}{2} \phi \alpha^{*2} \sigma^2 + \frac{1}{\phi} \ln(-\bar{\Pi}^T). \quad (34)$$

Since the tenant is by construction indifferent between the two contractual forms because his participation constraint is binding in both cases, the landlord will prefer a sharecropping contract to a fixed rental contract when  $0 < E[\Pi^{L*}] - R^{**}$ . By two trivial applications of the Envelope Theorem, the following PROPOSITION is then immediate:

**Proposition 3** *The landlord's preference for a sharecropping contract over a fixed rental contract is: (i) increasing in the variance  $\sigma^2$  of the stochastic shock affecting output, (ii) increasing in the degree of absolute risk-aversion  $\phi$  of the tenant.*

PROOF: see Appendix.

The intuition behind these two results should be obvious. As the variance of risk increases, the transfer that the tenant furnishes the landlord under the fixed rental contract falls in order for his participation constraint to remain satisfied. Similarly, in the case of the sharecropping contract, the magnitude of the transfer achieved through the share of output and the share of costs borne by the tenant also fall. However, the first fall (under fixed rental) is greater than the second (under sharecropping), since the cost to the tenant in terms of his welfare induced by a given increase in the variance of risk is greater under the fixed rental contract, where the tenant is the residual claimant, than in the sharecropping contract, where he only bears a fraction  $\alpha^*$  of the risk. Similarly, the more risk-averse the tenant, the greater the likelihood that the landlord will offer him a sharecropping contract because the costlier it will be (to the landlord) to induce him to accept residual claimancy.

The preceding setup is readily extended to allow for the landlord's decision to rent out or cultivate the plot himself as an owner-operator (whence the superscript  $O$ ). When the landlord cultivates the plot himself, his objective function, evaluated at the optimum, is given by:

$$E[\Pi^{O*}] = q^{***} - \sum_{i=1}^{i=I} X_i^{***} - \xi e^{***}, \quad (35)$$

where  $\xi$  is the landlord's opportunity "price" of effort (as with  $\omega$  for the tenant),

$$(X^{***}, e^{***}) = \arg \max_{\{X, e\}} F(T, X, e) - \sum_{i=1}^{i=I} X_i - \xi e, \quad (36)$$

and  $q^{***} = F(T, X^{***}, e^{***})$ . In light of our discussion concerning the choice between sharecropping and fixed rental contracts, it is then obvious that the landlord decides to farm the plot himself when:

$$E[\Pi^{O*}] = \max[E[\Pi^{O*}], R^{**}, E[\Pi^{L*}]]. \quad (37)$$

## 3 Empirical Implementation

### 3.1 A first pass

#### 3.1.1 The data

The data used in this paper stem from fieldwork undertaken by two of the authors in the village of El Oulja, Tunisia. See Matoussi and Nugent (1989), Laffont and Matoussi (1995) and Arcand, Ai, and Ethier (1998) for other descriptions of the village. Summary statistics for the full sample are provided in Tables 1 and 2 and will be discussed as needed in what follows.

At the outset, it is important to note that, since all of the data stem from households that live in one village, there is *no* cross-sectional variation in input prices. However, roughly one quarter (120) of the 455 plots in our sample are farmed under either sharecropping (45 plots) or fixed rental contracts (75 plots) with the sharecropped plots involving cost-sharing.<sup>11</sup> This allows us to identify the parameters of interest of the production technology through its usual dual representation in terms of the restricted profit function, because heterogeneity in cost-shares induces variation in

<sup>11</sup>We confine our attention to plots on which non-tree crops (olive and fruit trees are therefore excluded) are grown. Tree crops are subject to different, often intertemporal, contractual structures that are beyond the scope of this paper.

the *effective* price faced by the households cultivating the plots of land. Slightly more than one half of the cost-shares ( $\beta_i$ ) are equal to 50%, one third 100%, one sixth 0%, with the remainder being equal to 75, 70 or 66%.

That there is sufficient variation in cost-shares for one to be able to identify the relevant price effects in our empirical application is made clear in the statistics presented in Table 2. Here, it becomes obvious that there is substantial variability in the cost-shares, even when they are expressed as deviations with respect to operator household-specific means.<sup>12</sup> In the last three columns of Table 1, we report descriptive statistics for the three categories of plots. Sharecropped plots are, on average, larger than plots under fixed rental, which are in turn larger than owner-operator plots. Another important distinguishing feature of sharecropped plots is that they are less likely to be irrigated (the difference is significant at the 2% level). The only crop that is significantly less likely to be grown on sharecropped plots is fodder ( $p$ -value= 0.03).

In Tables 3 and 4, we divide land-owning and tenant households into different categories corresponding to their position on the land rental market. Land owners come in three basic varieties: pure owner-operators, pure landlords (who do not farm any of their land themselves), and mixed owner-operator/landlords (who farm some of the plots in their possession themselves and rent out the rest). We also single out landlords who are not full-time residents of the village. Since the landlords in question often live in a nearby village (Medjez-El-Bab), the term "non-resident" should be understood in this sense: many of these land owners are not absentee landlords by any means, though some, who reside in Tunis, are. Similarly, landlords who are not peasants themselves (again, this means that these landlords do not consider agriculture to be their *principal* activity, though it may occupy a good deal of their time) are considered separately. For tenants, we distinguish between those who do not farm any land of their own, and those who are owner-operators and simultaneously rent in.

The main features distinguishing pure landlords from pure tenants are household size and land ownership (see Table 4). Pure tenants correspond to large households of ten members (mixed operator tenants also correspond to larger households), while pure landlords correspond to households less than half this size. Conversely, pure tenants own very little land (4.33 hectares), which they do not farm themselves (often this is grazing land left fallow or olive trees, which are excluded from our sample), while pure landlords own four times as much land and simultaneously possess very little agricultural machinery (one of the reasons they do not operate the plots themselves) and are poorer. The pure landlords in El Oulja are therefore not the agricultural capitalists sometimes depicted as constituting the landlord side in the sharecropping literature; they are also more dependent on non-agricultural income (and therefore more likely to have a principal activity other than agriculture) than pure owner operators and mixed operator landlord households.

The role of agricultural capitalists is played in El Oulja by non-resident landlords, who also farm a substantial number of plots as owner-operators, because of their important stock of agricultural machinery. They also have very important land holdings and are significantly more educated on average (this difference in years of education stems from a number of landlords who we tracked down in Tunis and who are also urban professionals). As shown in Table 3, the mix of their rented out plots between sharecropping and fixed rental is slightly more skewed towards fixed

<sup>12</sup> Approximately one half of the standard deviation of a given cost share parameter stems from within-operator household heterogeneity. For example, for chemical fertilizer, the total standard deviation of the associated cost share parameter is 14.32, with the within operator household portion being equal to 8.94.

rental than for other categories of landlords, though this difference is not statistically significant. Non-peasant landlords, for their part, possess little agricultural machinery. One particularity of the sharecropping contracts to which non-resident landlords are a party is that they involve greater subsidization on their part of plowing, family labor, and especially hired labor (see Table 3). Seen from the tenant side, mixed tenant operators benefit from a greater level of cost-sharing for plowing than their pure tenant counterparts.

Mixed operator landlords differ from pure owner operators again in terms of their relative endowments of land and labor: the former own more land and come from slightly smaller households than the latter; they also possess less agricultural machinery. Finally, note that the heads of pure tenant households are significantly younger than the heads of all of the land owning categories, with mixed land owner tenants lying somewhere in between: this might be interpreted as evidence of some form of agricultural "ladder". Pure tenants are also much more involved in livestock than other types of households.

### 3.1.2 Naive specifications

In the village, the mean difference in the value of log output per hectare between sharecropped and other plots is equal to  $-0.564$  with a standard error of  $0.36$  ( $p$ -value =  $0.118$ ). If one compares sharecropped plots with those under fixed rental contracts, the corresponding difference is  $-1.033$  with a standard error of  $0.46$  ( $p$ -value =  $0.028$ ). Once one controls for soil type, irrigation status, and plot size, these differences are no longer statistically significant at the usual levels of confidence, though the sharecropped-fixed rental differential ( $-0.709$ , s.e. =  $0.42$ ) remains larger than the sharecropped *versus* all other plots differential ( $-0.187$ , s.e. =  $0.34$ ).

In column 1 of Table 5, we present an initial estimate of the difference in the logarithm of the value of output per hectare induced by sharecropping while accounting for operator household-specific effects.<sup>13</sup> This is essentially the procedure followed by Bell (1977) and Shaban (1987), as applied to the supply equation given in (20), where the vector  $(\alpha, \beta)$  has been collapsed into a "sharecropped plot" dummy, that takes on the value of one when the plot is farmed under a sharecropping contract, and zero otherwise. The specification includes a set of plot controls (4 soil type dummies, an irrigated plot dummy, 8 crop dummies), and log plot area.<sup>14</sup> In the interests of conciseness, we limit our presentation in the two Tables that follow to the coefficient associated with the sharecropped plot dummy. Our initial result suggests that there is no statistically significant difference in the value of output per hectare caused by sharecropping (the differential is  $-0.487$ , s.e. =  $0.53$ ), while controlling for unobserved operator household heterogeneity.<sup>15</sup> Moving

<sup>13</sup>We report results that control for household-specific, time varying unobservable heterogeneity (given that we have two time periods, over which we cannot, however, follow plots, since their definition changes each year). A more restrictive specification in terms of time-invariant household-specific effects yields similar results here, and in all that follows.

<sup>14</sup>The nine crops cultivated in the village are wheat, other grains, potatoes, onions, garden vegetables, tomatoes, beetroot, melon and fodder.

<sup>15</sup>We report  $t$ -statistics in parentheses in the Tables in order to facilitate their reading. In the absence of the crop dummies, the differential is  $-0.284$ , s.e.=  $0.55$ . One obtains roughly the same results —for all of the columns of Table 5, when one replaces the sharecropping dummy with the average value of  $\beta_i$  over all factor inputs, each individual  $\beta_i$  being weighted by its share in total costs. Note that while the null hypothesis of a zero differential is *not* rejected, this is not the same as saying that the absence of a differential *is* rejected. This is because the standard error associated with the coefficient on the sharecropped plot dummy reported in column 1 of Table 5 is sufficiently large that an Andrews (1989) inverse power function calculation indicates that a difference of  $-1.769$  *can* be confidently rejected (i.e., the test has high power against this alternative), while a difference of  $-0.884$  *cannot*

to household-specific *random* effects (which are not rejected by the appropriate Hausman test,  $\chi_{15}^2 = 14.560$ , whose  $p$ -value is 0.483) yields a differential of  $-0.461$ , but which remains statistically indistinguishable from zero ( $p$ -value= 0.165) at the usual levels of confidence.

In light of the theoretical discussion of part 2, it should be obvious that this first equation is grossly misspecified, first in terms of functional form (this is not a well-specified supply equation), and second in terms of the patent endogeneity of contractual choice. It is therefore not surprising that the ensuing results are inconclusive.

### 3.1.3 Instrumenting contractual choice

The theoretical model presented in part 2 of the paper suggests that there are two potential sources of instruments with which to endogenize contractual choice. Recall that optimal factor inputs (in their reduced form incarnation) are given by  $X^* = X^*(T, \omega, \bar{\Pi}^T)$ , where  $\bar{\Pi}^T$  is the reservation utility of the tenant, assumed to be a function of tenant and landlord characteristics. While one cannot instrument contractual choice using operator (tenant) characteristics since all variables are already expressed in terms of deviations with respect to operator specific means (the ensuing variables would be equal to zero by construction), one can use landlord characteristics, as long as we have enough rented out plots in the sample, and as long as a sufficient number of tenants interact with more than one landlord.

This is essentially an empirical issue in terms of the variance of landlord characteristics, once they are expressed as deviations with respect to operator specific means.<sup>16</sup> As should be clear from the summary statistics presented in Table 2, there are a sufficient number of tenants who engage in contractual relationships with more than one landlord for there to be substantial within operator household variation in landlord characteristics. In general, the within operator household standard deviation in landlord characteristics accounts for more than one half of the total standard deviation of the variables in question. Despite this promising aspect of the data in terms of within operator household variation in the potential instruments, we shall be particularly sensitive to "weak instrument" concerns and subject our statistical procedures to a number of tests designed to assess the "strength" of our proposed instruments.

In column 3 of Table 5 we present IV estimates of the impact of sharecropping on output per hectare, while controlling for operator household-specific effects, in which contractual choice is instrumented using within operator household variation of twelve landlord characteristics: non-agricultural income, landownership, residency status, occupational status (peasant or not), years of schooling, net wealth, household size, prime-age females in household, value of livestock, value of agricultural machinery, age, and pension income. All other aspects of the specification are identical to the within estimation results presented in column 1 of the same Table.<sup>17</sup>

As with the within results, the point estimate and its associated  $t$ -statistic indicate that there

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(the test has low power against this alternative), and a differential of 88% in the value of output per hectare as a result of sharecropping is certainly not inconsequential. The use of inverse power functions was suggested to us by Hanan Jacoby, whom we thank.

<sup>16</sup>This is particularly true in that, for owner operator plots, in which the tenant and the landlord are one and the same, the proposed instruments are zero by construction.

<sup>17</sup>In our empirical work, we experimented widely with different combinations of our instrumental variables, including much more narrowly focused sets of IVs. We restrict our discussion here to an instrument set that is broad enough to allow one to instrument all of the cost-sharing elements of the contract individually in section 3.3 of the paper, so as to render the results here and those reported later broadly comparable.



is no statistically significant impact of sharecropping on output per hectare. This is true whether one considers conventional 2SLS estimation, the bias-adjusted 2SLS (B2SLS) Nagar estimator suggested by Donald and Newey (2001), limited information maximum likelihood, or the Fuller (1977) estimator (with the "Fuller constant" set equal to 1).

Although these results are broadly in line with the picture painted by the within procedure, they must be handled with caution in that the test of the overidentifying restrictions soundly rejects, be it based on conventional 2SLS or on the LIML/Fuller estimates. Since it is well-known that tests of overidentifying restrictions tend to be erratic in the context of weak instruments—in particular the size of the test may be significantly larger than its nominal value—it is to this issue, and to a number of recent tests designed to assess the validity of instrumental variables in such a context, that we now turn.

## 3.2 Robustness

### 3.2.1 The weak instruments null

As is by now well known, weak instruments can lead to severe bias in IV estimation, and this bias does not vanish even with large sample sizes.<sup>18</sup> As a result, much effort has recently been devoted to finding ways of diagnosing situations in which weak instruments may be a concern. The standard response up until now has been to present the Shea (1997)  $R^2$  and  $F$ —statistics from the "partialled out" reduced form.

As should be clear from the values reported in column 3 of Table 5, our proposed instruments do not appear to be weak in that both the  $R^2$  and  $F$  statistics are above the usual critical values (the standard cutoff value of the partial  $F$  statistic is usually held to be around 10). Note however, that recent work, such as Cruz and Moreira (2005), has shown that such diagnostic tests can be extremely poor indicators of instrument weakness. This leads us to another test, designed to simultaneously assess instrument orthogonality and "strength", and which has the additional advantage of being distributionally based.

### 3.2.2 The strong instruments null

The spirit of the Hahn and Hausman (2002a) approach is different from that of the  $R^2$  and  $F$ —statistic diagnostics, which are implicitly based on the null hypothesis of *weak* instruments. In contrast, Hahn and Hausman (2002a) base their procedure on the null of *strong* instruments.

Consider the B2SLS estimator, which is an example of a  $k$ -class estimator, of which conventional 2SLS, the Fuller (1977) estimator and LIML are special cases. As an illustration, consider the simple situation, as in the specification presented in column 3 of Table 5, in which there is one jointly endogenous right-hand-side (RHS) variable (the sharecropped plot dummy), denoted by  $y_2$ . Consider a structural equation of the form:

$$y_1 = y_2 b + \varepsilon, \tag{38}$$

in which all of the predetermined variables have been "partialled out," and  $\varepsilon$  is the disturbance

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<sup>18</sup>See the excellent surveys by Stock, Wright, and Yogo (2002) and Hahn and Hausman (2003), and a recent very short primer on the ensuing biases by Hahn and Hausman (2002b).

term. Then the  $k$ -class instrumental variables estimator for the parameter  $b$  is defined by:

$$b_{B2SLS} = \frac{y_2' P_Z y_1 - \lambda y_2' M_Z y_1}{y_2' P_Z y_2 - \lambda y_2' M_Z y_2}, \quad (39)$$

where  $Z$  is the  $K$ -dimensional matrix of excluded instruments (in column 3 of Table 5,  $K = 12$ ) and  $M_Z = I - Z(Z'Z)^{-1}Z' = I - P_Z$  is the idempotent "anihilator matrix". For  $\lambda = [(K-2)/n] / [1 - (K-2)/n]$  where  $n$  is sample size, we obtain the B2SLS estimator proposed by Donald and Newey (2001), whereas  $\lambda = 0$  corresponds to conventional 2SLS.<sup>19</sup>

The Hahn and Hausman (2002a) test for the validity of the instrumental variables is constructed by running the B2SLS regression in its usual "forward" form, and comparing the result to that obtained by running the "reverse" regression, in which the jointly endogenous RHS variable (the "share" dummy here) is moved to the LHS, and the dependent variable (log output per hectare here) is entered on the RHS. The reverse B2SLS estimator is given by:

$$b_{RB2SLS} = \frac{y_1' P_Z y_1 - \lambda y_1' M_Z y_1}{y_1' P_Z y_2 - \lambda y_1' M_Z y_2}. \quad (40)$$

The basis for their test is that, if the instruments are orthogonal to the disturbance term in the structural equation and if they are "strong", standard first-order asymptotics imply that there will be very little difference between the results one obtains using the forward ( $b_{B2SLS}$ ) or reverse regressions ( $b_{RB2SLS}$ ). The test, referred to as the  $m_2$  test statistic, is standardized by using a second-order expression for the variance of the difference between the forward and reverse estimators, and can be read as a simple  $t$ -statistic.<sup>20</sup> More formally,  $m_2 = \hat{d}_2 / \sqrt{\hat{w}_2}$  where  $\hat{d}_2 = \sqrt{n}(b_{B2SLS} - b_{RB2SLS})$ , and:

$$\hat{w}_2 = \frac{2(K-1)(n-1)^2 \sigma_{\varepsilon, LIML}^4}{(n-1)b_{LIML}^2 \left[ y_2' P_Z y_2 - \left( \frac{K-1}{n-K} \right) y_2' M_Z y_2 \right]^2}, \quad (41)$$

where  $b_{LIML}$  is the LIML estimate of  $b$ , and  $\sigma_{\varepsilon, LIML}^2$  is the variance of the residuals of the structural equation estimated by LIML.<sup>21</sup>

In column 3 of Table 5, we therefore report both the forward and reverse B2SLS estimates of the coefficient associated with the sharecropped plot dummy.<sup>22</sup> For our proposed instrument set,

<sup>19</sup>Note that B2SLS only becomes a meaningful alternative to 2SLS once the degree of overidentification is strictly greater than 1 since B2SLS is identical to 2SLS when  $K = 2$ .

<sup>20</sup>Asymptotic properties of the test are presented in Hausman, Stock, and Yogo (2004), and the Montecarlo evidence shows "that the weak-instrument asymptotic distributions provide good approximations to the finite sample distributions for samples of size 100." The B2SLS estimator was originally proposed by Nagar (1959), hence the name often ascribed to it.

<sup>21</sup>We do not present the  $m_1$  statistic based on forward and reverse 2SLS. Its use is no longer recommended, following a number of Montecarlo exercises that have been performed since publication of the original paper. We are grateful to Jerry Hausman for pointing this out to us. Note also that one can replace the LIML estimates of the nuisance parameters by their Fuller estimator counterparts. This does not lead to any appreciable quantitative differences in our results.

<sup>22</sup>Note that the LIML estimate is expected according to theory to fall between the forward and reverse B2SLS results because it is an optimal linear combination of these two estimators: this is not the case in the results presented in Table 5 (it *will* be in those presented in Table 6), which constitutes another informal indication of problems with the specification. The use of Bekker (1994) standard errors, along with the Fuller estimator, does not change the results of our statistical inference. We also estimated these equations using the Jackknife instrumental variables estimator (JIVE) proposed by Angrist, Imbens, and Krueger (1999) and for which some simulation evidence exists (see Blomquist and Dahlberg (1999), who also present simulation evidence in the weak instruments case for LIML).

the four "forward" point estimates (2SLS, B2SLS, LIML and Fuller) differ by very little, whereas the reverse-B2SLS estimate is strikingly different ( $-44.042$ ) and is highly implausible in economic terms. As such, it is not surprising that the  $m_2$  test statistic rejects the validity of our instrument set for the specification at hand ( $p$ -value = 0.035), and this despite the large standard errors associated with the forward and reverse B2SLS estimates. The upshot is that our earlier rejection of the overidentifying restrictions is confirmed, and that this may be caused by an overly restrictive specification.

### 3.2.3 Plot size, the decision to rent out, and cropping

Given our earlier theoretical discussions, three problems potentially plague the simple specification presented in column 3 of Table 5. First, we have assumed that plot area is exogenous, whereas it may be set by the landlord along with the other terms of the contract, as described formally in PROPOSITION 1(v).

Second, we have not explicitly considered the landlord's decision to rent out the plot rather than keeping it under owner-operatorship, as spelled out in section 2.6 and in equation (37). In the specifications considered so far, and through our focus on the sharecropped plot dummy alone, we have implicitly grouped owner-operatorship and fixed rental into one outcome (coded as zero), with sharecropping constituting the second outcome (coded as one).<sup>23</sup> A potential problem, suggested in part by the theoretical work on the choice among owner-operatorship, sharecropping and fixed rental, is that the decision to rent out may be distinct from the choice between sharecropping and fixed rental, and will depend upon landlord characteristics. Among other mechanisms, the decision to rent out may be driven by heterogeneity in the landlords' marginal disutility of effort  $\xi$ . A further problem is that residual claimancy in and of itself may not constitute a sufficiently rich characterization of the incentives facing the tenant, and cultivating a rented out plot *per se* may induce incentive effects on its own. Moreover, rented out plots as a whole may possess unobserved characteristics that are not adequately captured by our plot-level controls. As such, one should condition identification of the impact of sharecropping on the plot being rented out, through inclusion (and potential instrumentation) of the rented out plot dummy.<sup>24</sup>

Third, the baseline specification ignores the fact that it may be the landlord who chooses the crop grown on the plot, as noted in the discussion in section 2.5 following PROPOSITION 2, and as set out explicitly in equation (31). In order to endogenize this choice in a manner that preserves degrees of freedom, which will be important later on when we will have to instrument each term in

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The JIVE1 estimator is given by adding

$$Z_i \hat{\pi}(i) = \frac{Z_i \hat{\pi} - h_i y_{2i}}{1 - h_i}$$

as an instrument, where  $i$  indexes each observation ( $i = 1, \dots, n$ ),  $\hat{\pi}$  is the reduced form parameter vector estimated using all observations, and  $h_i$  is the "leverage" of observation  $i$ . The JIVE2 estimator replaces  $1 - h_i$  with  $1 - \frac{1}{n}$  in the denominator. Results were rather erratic (and depended on whether one used JIVE1 or JIVE2), which may be linked to the problems associated with this estimator pointed out by Davidson and MacKinnon (2004).

<sup>23</sup> Abstracting from the rare cases where cost-sharing obtains in the case of fixed rental, our alternative specification (which we do not report because the results are similar to our baseline results), in which the sharecropping dummy is replaced by the average value of the cost-shares, corresponds to the same grouping.

<sup>24</sup> One alternative that we investigated was a nested logit model of the decision to rent out followed by contractual choice. The estimated hazard rate corresponding to the decision to rent out was then included in the second stage as an additional regressor. We then either instrumented this hazard rate directly, using our usual instrument set, or bootstrapped the standard errors in order to account for the generated nature of the additional explanatory variable. Results, in terms of the impact of sharecropping on output per hectare, are not sufficiently different to warrant their presentation, and we prefer the more direct route of instrumenting both dummies directly, as presented in the text.

$(\alpha, \beta_1, \beta_2, \dots, \beta_i, \dots, \beta_I)$  separately, we begin by running a multinomial logit on the nine potential crop choices, as a function of landlord characteristics. The fit of this equation is quite good, with a scaled  $R^2$  of 0.640 and a likelihood ratio test of the zero coefficients null hypothesis that rejects with a  $p$ -value below 0.001. As in Thomas and Strauss (1997), in the context of sectoral choice in a Mincerian wage equation, the ensuing hazard rate is then inserted into our structural equation, and is allowed to be jointly endogenous.

In passing, it is important to note that collapsing the eight crop choice dummies into a scalar hazard rate does not change the gist of our earlier results. For example, in the within results reported in column 2 of Table 5, where we replace the eight crop dummies included in column 1 with the hazard rate stemming from our multinomial logit model of crop choice, little changes with respect to our earlier results. The same is true for the IV results reported in column 4 of Table 5 (compare this with the results in column 3), where the sharecropping dummy remains statistically indistinguishable from zero, the overidentifying restrictions continue to be rejected, and the Hahn-Hausman test also rejects.

Before proceeding to the estimation results, it is important to note that the Hahn-Hausman test statistic must be modified when one faces more than one jointly endogenous RHS variable. More specifically, when there are  $r > 1$  endogenous variables present in the structural equation, Hahn and Hausman (2002a) show that there are  $r - 1$  different reverse regressions that can be run but that *no* gains in efficiency are achieved by stacking the various parameter estimates obtained through these different normalizations: the best that one can do is to use *one* difference between forward and reverse results. If we arbitrarily consider the reverse regression in which  $y_2$  is put on the LHS and  $y_1$  on the RHS, with  $y_j, j \geq 3$  denoting the other endogenous RHS variables, then the  $m_3$  test statistic is given by  $m_3 = \hat{d}_3 / \sqrt{\hat{w}_3}$  where  $\hat{d}_3 = \sqrt{n}(b_{2,B2SLS} - b_{2,RLIML})$ , and:<sup>25</sup>

$$\hat{w}_3 = \frac{2 \left( \frac{K-1}{n-K} \right) \left[ \sum_{i=1}^{i=n} \left( y_{1i} - \sum_{j=2}^{j=r} b_{j,LIML} y_{ji} \right)^2 \right]^2}{\beta_{2,LIML}^2 \left[ \frac{y_2' P_Z y_2 - \left( \frac{K-1}{n-K} \right) y_2' M_Z y_2}{-\sum_{j=3}^{j=r} \frac{(y_j' P_Z y_j - \left( \frac{K-1}{n-K} \right) y_j' M_Z y_j)^2}{y_j' P_Z y_j - \left( \frac{K-1}{n-K} \right) y_j' M_Z y_j}} \right]^2}. \quad (42)$$

Results corresponding to the least restrictive specification, in which the sharecropped plot dummy, plot size, the rented out plot dummy, and crop choice are all allowed to be endogenous are presented in column 1 of Table 6. As with our earlier results, there is no statistically significant difference in the value of output per hectare induced by sharecropping. In contrast to our earlier results, on the other hand, the overidentifying restrictions are *not* rejected at the usual levels of confidence, the difference between the forward and reverse B2SLS estimates is reduced, and the Hahn-Hausman  $m_3$  test of instrument validity does *not* reject.

The differences between the 2SLS estimate and its B2SLS, LIML or Fuller counterparts suggest that finite sample problems bias the 2SLS estimates upwards, though all of the forward estimates paint a similar picture which is, moreover, substantiated by our two diagnostic tests of instrument validity: although Arthur Young may have noted (as quoted by Alfred Marshall (1920), p. 537),

<sup>25</sup>This formula is a simple generalization of that given in Hahn and Hausman (2002a), equation 9.4.

that "the magic of property turns sand into gold," sharecropping is not associated with a lower level of output per hectare in this Tunisian village, *ceteris paribus*.<sup>26</sup>

### 3.2.4 Instrument choice

In an effort to isolate the most important endogeneity concerns that affect our estimates, and potentially arrive at a more parsimonious specification in terms of exclusion restrictions, we resort to a diagnostic tool designed to identify an "optimal" instrument set. In doing so, we restrict our attention to those specifications that are rejected neither by the test of the overidentifying restrictions, nor by the Hahn-Hausman test.

Our diagnostic tool is the Andrews (1999) instrument selection procedure, which is based on the Bayesian (BIC), Akaike (AIC) and Hannan-Quinn (HQ) model selection information criteria. These tests are based on the  $J$  test statistic for the over-identifying restrictions, from which one subtracts a "bonus term" that rewards instrument sets that use more exclusion restrictions. Though the Andrews test is in principle geared towards weeding out invalid instruments, the author suggests that one should limit its application to small sets of potential instruments. The instrument set which minimizes the instrument selection criteria (IV-BIC, IV-AIC and IV-HQIC) is then the preferred choice.

In columns 2 and 3 of Table 6, we report results for the two other specifications for which instrument validity is not rejected. In column 2, the rented out plot dummy is allowed to be exogenous, whereas in column 3, both the rented out plot dummy and plot size are assumed to be exogenous. The sharecropping dummy and the crop choice hazard rate remain jointly endogenous in both specifications. On the basis of all three versions of the Andrews test, reported in the lower portion of Table 6, our choice should fall on the specification reported in column 3. It is clear, on the basis of these tests, that sharecropping and crop choice must be considered as being endogenous, whereas plot size and the rented out dummy can be assumed exogenous. For all three specifications presented in Table 6, however, there is no statistically significant impact of sharecropping on output per hectare, and the point estimates are all relatively similar, for a given forward IV estimator. At the heuristic level, our preference for the results reported in column 3 is strengthened by the fact the forward and reverse B2SLS point estimates are quite close.<sup>27</sup>

<sup>26</sup>An alternative specification that we investigated was a panel IV procedure on the subsample of rented out plots, which corrects for selection bias, as suggested recently by Wooldridge and Semykina (2005) (the basic ideas are set out in Wooldridge (1995)). Note that applying a Heckman procedure using operator household specific effects would result in an inconsistent estimate of the impact of sharecropping. In the first stage of the procedure, we estimate a probit on the decision to rent out, as a function of our instrument set (including the exogenous included variables) and, applying the usual Mundlak (1978) device, our instruments *expressed as operator-specific means*. In the second stage of the procedure, we estimate our structural equation on the subsample of rented out plots by *pooled* 2SLS, where the explanatory variables are (i) those included in our usual specification (the sharecropped plot dummy and the cropping hazard rate are allowed to be jointly endogenous), (ii) the instruments, *expressed in terms of operator-specific means*, and (iii) the inverse Mills ratio from the first stage probit. The excluded IVs are given by our usual instrument set. The coefficient associated with the sharecropped plot dummy is equal to  $-0.416$  with a  $t$ -statistic of  $-0.35$ , and the test of overidentifying restrictions and the Hahn-Hausman test do not reject, confirming our results based on the full sample. Given the structure of our data, a pairwise-differencing sample selection estimator, such as Kyriazidou (1997), is not feasible here.

<sup>27</sup>We also carried out the Donald and Newey (2001) "choice of instruments" test. This test is based upon choosing, from within a number of valid instrument sets, the one which minimizes mean-squared error (MSE). In the first stage of the test procedure, one identifies the instrument set that minimizes the Mallows (or cross-validation) reduced form goodness of fit criterion. This instrument set is then used to compute initial estimates of the variance of the reduced form and structural equation residuals, as well as their covariance, which enter into the expressions for the Mallows criterion and the MSEs. In the second stage of the procedure, one recomputes the Mallows criterion for each instrument set. This is then plugged into the appropriate expression for the MSE (see Donald and Newey,

### 3.2.5 Extensions

Our results remain unchanged when we replace the sharecropped plot dummy with the ten cost share-to-output-share ratios  $(\beta_1/\alpha, \beta_2/\alpha, \dots, \beta_i/\alpha, \dots, \beta_I/\alpha)$ , allow the crop choice hazard rate to be endogenous, and use the same instrument set. In this case, as shown by the summary results reported in the first row of Table 7, the  $p$ -value for the  $\chi^2_{10}$  test of the joint significance of the  $\beta_i/\alpha$  ratios is equal to 0.362 for 2SLS, and is equal to 0.293 for the Fuller estimator (none of the  $\beta_i/\alpha$  ratios are individually significant for any of the forward IV estimators). The  $p$ -value associated with the 2SLS-based test of the overidentifying restrictions is equal to 0.328, 0.568 for its LIML/Fuller counterpart, while the Hahn-Hausman  $m_3$  test does not reject ( $p$ -value = 0.531).

Carrying out the same procedure for all ten factor inputs, expressed as logarithms of their value per hectare, shows that our set of excluded IVs, alongside the assumption of endogenous crop choice, is robust in terms of the validity of the underlying exclusion restrictions, since none of the tests of the overidentifying restrictions reject, at the usual levels of confidence.<sup>28</sup> Note however, that the Hahn-Hausman test rejects for chemical fertilizer, manure, plowing, hired labor, seeds and transportation: taken in conjunction with the fact that the overidentifying restrictions are not rejected, by a wide margin, for any of these factor inputs, this suggests that our instruments, while orthogonal to the structural equation's disturbance term, are potentially weak for these six factor input equations.

As such, it would appear to be preferable to base one's inference on the Fuller estimates, since this estimator possesses finite moments for all values of the "concentration parameter" associated with the reduced forms, as well as good small sample properties. Moreover, Hahn, Hausman, and Kuersteiner (2004) have provided extensive Monte Carlo experiment results that show that this estimator performs well when compared to other prominent IV estimators, under weak instruments. On the basis of the Fuller-based tests of the joint significance of the  $\beta_i/\alpha$  terms, one would then reject Marshallian efficiency for irrigation and transportation, using a five percent critical level, while continuing to find no statistically significant impact of sharecropping on output per hectare.

## 3.3 Applying the duality mapping

### 3.3.1 Choosing the functional form

In order (i) to circumvent the empirical problems that plague current tests of Marshallian efficiency, discussed in section 2.3, and (ii) to implement the formal test of contractual optimality provided by PROPOSITIONS 1 and 2 in a manner that is consistent with the heterogeneity in cost-shares observed in the data, careful thought must be given to the parametric form taken by the production technology. Moreover, and as we have stressed earlier, results such as those presented in Tables

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2001, pp. 1164-5), which itself depends upon the choice of instrument set. The instrument set which minimizes the MSE is then the one that should be used. The instrument set which includes the rented out plot dummy and plot size minimizes the Mallows criterion, and this instrument set is therefore the one used to compute all variance and covariance terms that go into the MSE formulae. Moreover, it turns out that this instrument set also minimizes the MSEs of all three forward estimators considered (2SLS, B2SLS and LIML), and that the differences with respect to the two other instrument sets are quite large. The conclusion, on the basis of the Donald and Newey test, as with the Andrews procedure, is therefore that the specification presented in column 3 of Table 6 corresponds to the preferred exclusion restrictions, whether one uses 2SLS, B2SLS or LIML.

<sup>28</sup>One is very close to rejecting for the case of irrigation using the 2SLS-based test, but the  $p$ -value rises significantly when the test is based on the Fuller estimator

5, 6 and 7, while of some interest, fail to impose even the most elementary restrictions stemming from optimizing behavior by the peasants operating the plots of land.

In the present context, in which heterogeneity in the elasticities of substitution between factor inputs is crucial (see section 2.4.2), a natural choice is the translog restricted profit function (Christensen, Jorgenson, and Lau (1971), also see Christensen, Jorgenson, and Lau (1973)), which for the case at hand can be written as:<sup>29</sup>

$$\begin{aligned}
\ln \Pi^{T*} &= \delta_\alpha \ln \alpha + \sum_{i=1}^{i=I} \delta_i \ln \beta_i + \delta_\omega \ln \omega \\
&+ \gamma_{\alpha\omega} \ln \alpha \ln \omega + \frac{1}{2} \left[ \gamma_{\alpha\alpha} (\ln \alpha)^2 + \gamma_{\omega\omega} (\ln \omega)^2 \right] \\
&+ \sum_{i=1}^{i=I} \left( \begin{array}{c} \gamma_{i\alpha} \ln \beta_i \ln \alpha + \gamma_{i\omega} \ln \beta_i \ln \omega \\ + \frac{1}{2} \sum_{j=1}^{j=I} \gamma_{ij} \ln \beta_i \ln \beta_j \end{array} \right) \\
&+ \sum_{h=1}^{h=H} \left( \begin{array}{c} \zeta_h W_h + \zeta_{\alpha h} W_h \ln \alpha \\ + \zeta_{\omega h} W_h \ln \omega + \sum_{i=1}^{i=I} \zeta_{ih} W_h \ln \beta_i \end{array} \right),
\end{aligned} \tag{43}$$

where the "prices" are given by the  $I + 2$  dimensional vector  $(\alpha, \beta, \omega)$ , and  $W_h$  is a vector of  $H$  plot-level controls. Linear homogeneity in prices is ensured by the restrictions  $\delta_\alpha + \sum_{i=1}^{i=I} \delta_i + \delta_\omega = 1$ ,  $\zeta_{\alpha h} + \sum_{i=1}^{i=I} \zeta_{ih} + \zeta_{\omega h} = 0$ ,  $h = 1, \dots, H$ ,  $\gamma_{\alpha j} + \sum_{i=1}^{i=I} \gamma_{ij} + \gamma_{\omega j} = 0$ ,  $j = 1, \dots, I$ ,  $\gamma_{\alpha\alpha} + \sum_{i=1}^{i=I} \gamma_{i\alpha} + \gamma_{\omega\alpha} = 0$ ,  $\gamma_{\alpha\omega} + \sum_{i=1}^{i=I} \gamma_{i\omega} + \gamma_{\omega\omega} = 0$ , whereas  $\gamma_{ij} = \gamma_{ji}$ ,  $\forall i \neq j$ , yields the symmetry property.<sup>30</sup>

The well-known advantage of the translog form is that the factor share equations are, by Hotelling's Lemma, linear. The translog technology does not impose constant elasticities of substitution among factor inputs as do the Cobb-Douglas and CES functional forms; it is therefore compatible with heterogeneity in optimal cost-shares as defined by PROPOSITIONS 1 and 2. It is, moreover, a "flexible" functional form in that it represents a second-order differential approximation to an arbitrary restricted profit function.<sup>31</sup>

### 3.3.2 Obtaining an estimable specification in terms of observables

Note that, empirically speaking, our data pertain to elements of the "observable profit function":

$$\ln \tilde{\Pi}^{T*} = \ln (\Pi^{T*} + \omega e^*) = \ln \Pi^{T*} + \ln (1 + s_\omega), \tag{44}$$

where  $s_\omega = \frac{\omega e^*}{\Pi^{T*}}$ , whereas the specification given in (43) corresponds to the *unobservable* profit function that includes the opportunity cost of effort. It is therefore crucial, for one to be able to obtain empirical versions of the producers theory-based arguments presented in part 2, to establish a mapping between the profit function and its observable counterpart. Once again, the restrictions provided by duality theory allow one to do so.

<sup>29</sup>Another possibility that was explored in the empirical work was the Generalized Leontief restricted profit function, initially proposed by Diewert (1971) (also see Diewert (1974) and Blackorby and Diewert (1979)). Both the translog and the Diewert restricted profit functions are examples of flexible functional forms (Diewert (1974), Lau (1974)).

<sup>30</sup>The symmetry restrictions  $\gamma_{\alpha j} = \gamma_{j\alpha}$ ,  $\gamma_{\omega j} = \gamma_{j\omega}$ ,  $j = 1, \dots, I$  are already imposed in the specification presented in (43).

<sup>31</sup>In what follows, we assume that the production technology is the same under all contractual forms. This assumption has sometimes been questioned in the literature on sharecropping.

Applying Hotelling's Lemma to (43) yields the translog "share" equation for the value of effort, which is linear in log prices:

$$s_\omega = \delta_\omega + \gamma_{\alpha\omega} \ln \alpha + \gamma_{\omega\omega} \ln \omega + \sum_{i=1}^{i=I} \gamma_{i\omega} \ln \beta_i + \sum_{h=1}^{h=H} \zeta_{\omega h} W_h. \quad (45)$$

Substituting (45) into (44), it follows that:

$$\ln \tilde{\Pi}^{T*} = \ln \Pi^{T*} + \ln \left( 1 + \sum_{h=1}^{h=H} \zeta_{\omega h} W_h + \delta_\omega + \gamma_{\alpha\omega} \ln \alpha + \gamma_{\omega\omega} \ln \omega + \sum_{i=1}^{i=I} \gamma_{i\omega} \ln \beta_i \right), \quad (46)$$

where  $\ln \Pi^{T*}$  is understood (for brevity's sake) to be given by the expression on the RHS of (43). The LHS of (46) is observable, while all of the variables on the RHS are observable, save for  $\ln \omega$ , which appears both in the term in parentheses and in the expression for  $\ln \Pi^{T*}$  from (43);  $\ln \omega$  is, however, common to all plots farmed by a given household and can therefore be captured by operator household-specific effects. Note that these do *not* appear only in additive form, because  $\ln(1 + s_\omega)$  is *not* linear in  $\ln \omega$ . This non-linearity is inescapable and constitutes the price that must be paid in order to correctly account for the opportunity cost of unobservable effort.

Applying Hotelling's Lemma to factor input  $i$ , followed by similar manipulations, yields:

$$\ln(\beta_i X_i^*) = \ln \Pi^{T*} + \ln \left( \sum_{h=1}^{h=H} \zeta_{ih} W_h + \delta_i + \gamma_{i\alpha} \ln \alpha + \gamma_{i\omega} \ln \omega + \sum_{j=1}^{j=I} \gamma_{ij} \ln \beta_j \right), \quad (47)$$

whereas a final application of Hotelling's Lemma yields:

$$\ln(\alpha q^*) = \ln \Pi^{T*} + \ln \left( \sum_{h=1}^{h=H} \zeta_{\alpha h} W_h + \delta_\alpha + \gamma_{\alpha\alpha} \ln \alpha + \gamma_{\alpha\omega} \ln \omega + \sum_{i=1}^{i=I} \gamma_{i\alpha} \ln \beta_i \right). \quad (48)$$

Note that under the assumptions of PROPOSITION 2 (i.e., CARA preferences and an additive, normally distributed shock to output), these estimating forms remain unchanged under tenant risk-aversion, since  $\ln \Pi^{T*}$  is the same.<sup>32</sup>

### 3.3.3 Symmetry, homogeneity and endogenous contractual choice

In the results presented in Table 8, we impose the symmetry and homogeneity properties that must be satisfied by any true system of factor demand and supply equations, by adopting the translog functional form and jointly estimating (46), the  $I$  equations defined by (47) for each physical factor input, and (48). The excluded "factor share equation" from producers theory is the one corresponding to unobservable effort. Estimation is by non-linear SUR (this yields a total of 12 equations), with all of the cross-equation restrictions that flow from duality theory. The fixed factors represented by  $W_h$  are log plot area, four soil type dummies, the irrigated plot dummy, the rented out plot dummy and the crop choice hazard rate discussed earlier.<sup>33</sup> Standard errors are

<sup>32</sup>Formally, what changes, because of the characterization of the optimal sharecropping contract, and when one wishes to endogenize it, is that the excluded instruments for  $(\alpha^*, \beta^*)$  must account for tenant risk-aversion ( $\phi$ ) and the variance of the stochastic shock ( $\sigma^2$ ), though this causes no additional empirical complications.

<sup>33</sup>For brevity of exposition, we only present estimates corresponding to the  $\delta_\alpha, \delta_i, \gamma_{\alpha i}$  and  $\gamma_{ij}$  parameters. Similarly, we do not report the coefficients associated with the plot-level controls since our focus is on the incentive



computed using the Huber-White formula.

Note that this specification is essentially the same as a set of "Shaban regressions" (in the sense that it controls for unobserved operator household-specific heterogeneity, while contractual structure is assumed exogenous), but where all of the restrictions from received producers theory have been imposed. In particular, duality theory allows one to account for unobservable effort in a manner that is consistent with optimal input choice on the part of the household cultivating the plot. In order to facilitate reading of the table, we only report parameter estimates that are significant at the 5% level or better: the *empty* spaces in Table 8 therefore correspond to coefficients that are estimated, but whose associated  $p$ -value is above 0.05.

Were one to base one's conclusions regarding Marshallian inefficiency on this specification using the standard Shaban methodology (i.e. based on the statistical significance of the contract variables –the  $\gamma_{ij}$  coefficients in the factor demand and output equations), one would reject Marshallian efficiency for *all* inputs (visually, there are no factor inputs for which both the corresponding line and column of the  $\gamma_{ij}$  coefficient portion of the table are empty). Of course, these results must be handled with caution in that they are based on the maintained hypothesis that contractual choice, as well as crop choice, are exogenous.

In Table 9, we endogenize contractual choice and crop choice in the same manner as in the simple output per hectare equation results presented earlier, using the same instrument set of *landlord* characteristics, expressed as deviations with respect to *operator*-specific means. As with the empirical results presented in section 3.2.5, the endogeneity of contractual structure is modelled by allowing the vector  $(\ln \alpha, \ln \beta_1, \ln \beta_2, \dots, \ln \beta_i, \dots, \ln \beta_I)$ , as well as the crop choice hazard rate, to be jointly endogenous. In this case, estimation is performed using a minimum distance estimator. It should be noted that non-negligible finite sample bias may be present in the results presented in Table 9 in that the Hahn-Hausman tests carried out in section 3.2.5 revealed that six out of ten "naive" factor input equations were subject to weak instrument concerns, and this problem is likely to affect the correctly specified equations that underly Table 9.

Subject to this *caveat*, and under the specification presented in Table 9, one would therefore reject Marshallian efficiency –again based on the Shaban approach– for all factor inputs, as well as for output. This is in stark contrast to the results presented in Table 6, in which Marshallian efficiency was not rejected for output, and to those presented in Table 7, in which Marshallian efficiency was only rejected for two factor inputs.

### 3.4 Are the sharecropping contracts second-best optima?

Consider the characterization of the optimal cost-sharing contract given in PROPOSITIONS 1 or 2. Given our estimates of the production technology presented in the previous section, these restrictions can be directly tested. In order to gain a better intuitive understanding of the importance of the choice of the production technology in testing the optimality of contractual structure, note that the translog technology implies that:

$$\frac{dX_i^*}{d\beta_j} = -(\gamma_{ij} + s_i s_j) \frac{\Pi^{T*}}{\beta_i \beta_j} = -\sigma_{ij} \frac{s_i s_j}{\beta_i \beta_j} \Pi^{T*}, \quad (49)$$

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effects of the sharecropping contracts.

where  $\sigma_{ij} = \frac{\gamma_{ij} + s_i s_j}{s_i s_j}$  is the Allen-Uzawa elasticity of substitution (AUES) between factor inputs  $i$  and  $j$ , when  $i \neq j$ , and  $s_i = \frac{\beta_i X_i^*}{\Pi^{T*}}$ , whereas:

$$\frac{dX_i^*}{d\beta_i} = -(\gamma_{ii} + s_i + s_i^2) \frac{\Pi^{T*}}{\beta_i^2} = -\sigma_{ii} \frac{s_i^2}{\beta_i^2} \Pi^{T*}, \quad (50)$$

where  $\sigma_{ii} = \frac{\gamma_{ii} + s_i + s_i^2}{s_i^2}$ .<sup>34</sup> Similarly,  $\frac{de^*}{d\beta_i} = -(\gamma_{i\omega} + s_i s_\omega) \frac{\Pi^{T*}}{\beta_i s_\omega} = -\sigma_{i\omega} \frac{s_i s_\omega}{\beta_i s_\omega} \Pi^{T*}$ , while  $\frac{dq^*}{d\beta_i} = (\gamma_{i\alpha} - s_\alpha s_i) \frac{\Pi^{T*}}{\alpha \beta_i} = -\sigma_{i\alpha} \frac{s_\alpha s_i}{\alpha \beta_i} \Pi^{T*}$ , where  $\sigma_{i\alpha} = \frac{\gamma_{i\alpha} - s_i s_\alpha}{-s_i s_\alpha}$  and  $s_\alpha = \frac{\alpha q^*}{\Pi^{T*}}$ . Simple algebraic manipulations then allow one to write PROPOSITION 1(i), for  $i = 1, \dots, I$ , as:

$$\begin{aligned} 0 = & \frac{\gamma_{i\alpha} - s_i s_\alpha}{\alpha^*} + \frac{\gamma_{ii} + s_i + s_i^2}{\beta_i^*} \\ & + \sum_{j=1, j \neq i}^{j=I} \left( \frac{\gamma_{ij} + s_i s_j}{\beta_j^*} \right) + \gamma_{i\omega} + s_i s_\omega. \end{aligned} \quad (51)$$

Similar expressions can be derived for the other parts of PROPOSITION 1, as well as for PROPOSITION 2. That optimal cost-shares are necessarily all equal in the case of unitary elasticity of substitution, as seen earlier, is then particularly transparent: it suffices to set all of the second order ( $\gamma$ ) terms in (51) equal to zero.<sup>35</sup>

Results of testing (51), for the median values over the 45 sharecropped plots, of the estimated  $s_i$ ,  $s_\alpha$  and  $s_\omega$ , as well as the medians of  $(\alpha^*, \beta_i^*)$ , are presented in the lower portion of Table 9. We also report the results of a similar test for the SUR specification in the lower portion of Table 8.

As should be clear from the results presented in Table 9, the restrictions given in (51) are rejected for all factor inputs, apart from plowing and hired labor. When the medians are replaced with means, the restrictions are rejected for all factor inputs except chemical fertilizer and manure (not reported). When we extend our test so that it also includes plots under fixed rental contracts, the factor inputs for which we do not reject are manure and seeds. In all cases, the joint null of the optimality of the cost-shares is rejected with an extremely low  $p$ -value.<sup>36</sup>

Despite the statistical significance of a large number of coefficients associated with contractual terms in all specifications, our results are therefore *not* consistent with optimization on the part of the landlord in a principal-agent framework, as set out in PROPOSITIONS 1 and 2. Based on our test results, the cost-sharing contracts implemented by landlords in El Oulja are therefore *not* second best optima. It remains that the share of various costs borne by the tenant are a significant determinant of factor input use.

## 4 Concluding remarks

This paper provides a bridge between two hitherto unrelated literatures: (i) moral hazard and contractual choice and (ii) duality approaches to the theory of the firm. This "missing link" was

<sup>34</sup>See, e. g. Nadiri (1982), pp. 467-8.

<sup>35</sup>This is because, setting  $\gamma_{i\alpha} = \gamma_{ii} = \gamma_{ij} = \gamma_{i\omega} = 0$ , the condition simplifies to  $\sum_{j=1}^{j=I} \frac{s_j}{\beta_j} - \frac{s_\alpha}{\alpha} + s_\omega + \frac{1}{\beta_i} = 0, i = 1, \dots, I$ , which then implies the equality of optimal cost-shares.

<sup>36</sup>This remains true when we add the restriction that must be satisfied by the optimal share of output (given by PROPOSITION 2(ii)).

then applied to that most traditional of development economics topics —Marshallian inefficiency, and highlighted a number of serious *lacunae* that persist in that literature despite several decades of research and several hundred published papers. While use of the standard duality mapping in the context of Marshallian inefficiency is arguably the main contribution of this paper, it remains that the impact on the empirical results of two elementary constraints that must be satisfied by any system of factor input demands (homogeneity and symmetry) is even more striking. Poor contract theory is bad. Poor microeconomic theory is worse.

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## A Appendix

**Proof.** PROPOSITION 1. The landlord's objective function is given by:

$$E[\Pi^L] = (1 - \alpha)q^* - \sum_{i=1}^{i=I} (1 - \beta_i)X_i^* \quad (52)$$

where we have substituted for  $(X^*, e^*)$  as determined by (16) and (17), and  $q^* = F(T, X^*, e^*)$ . Since the participation constraint is binding, it follows that:

$$E[\Pi^L] = q^* - \sum_{i=1}^{i=I} X_i^* - \omega e^* - \bar{\Pi}^T. \quad (53)$$

The optimal vector of cost-shares, denoted by  $\beta^*$ , is then defined by the following  $I$  FOCs (where we also let the other FOC,  $\frac{dE[\Pi^L]}{d\alpha} = 0$ , hold —more on this below):

$$\frac{dE[\Pi^L]}{d\beta_i} = \frac{dq^*}{d\beta_i} - \sum_{j=1}^{j=I} \beta_j \frac{dX_j^*}{d\beta_i} - \omega \frac{de^*}{d\beta_i} = 0, i = 1, \dots, I. \quad (54)$$

Now we know from standard producers theory that  $\Pi^{T*}$  is homogeneous of degree 1 in  $(\alpha, \beta, \omega)$ , while the supply function and the factor demand equations  $(q^*, X^*, e^*)$  are homogeneous of degree zero in  $(\alpha, \beta, \omega)$ . As is well-known, this implies, by Euler's Theorem and by the symmetry of the  $(I + 2) \times (I + 2)$  Hessian matrix associated with  $\Pi^{T*}$ , that:

$$0 = \alpha \frac{d^2 \Pi^{T*}}{d\alpha^2} + \sum_{j=1}^{j=I} \beta_j \frac{d^2 \Pi^{T*}}{d\alpha d\beta_j} + \omega \frac{d^2 \Pi^{T*}}{d\alpha d\omega} \quad (55)$$

$$= \alpha \frac{dq^*}{d\alpha} - \sum_{j=1}^{j=I} \beta_j \frac{dX_j^*}{d\alpha} - \omega \frac{de^*}{d\alpha},$$

$$0 = \alpha \frac{d^2 \Pi^{T*}}{d\alpha d\beta_i} + \sum_{j=1}^{j=I} \beta_j \frac{d^2 \Pi^{T*}}{d\beta_i d\beta_j} + \omega \frac{d^2 \Pi^{T*}}{d\beta_i d\omega} \quad (56)$$

$$= \alpha \frac{dq^*}{d\beta_i} - \sum_{j=1}^{j=I} \beta_j \frac{dX_j^*}{d\beta_i} - \omega \frac{de^*}{d\beta_i}, i = 1, \dots, I.$$

Grouping terms in equation (54) and using (56) then allows one to write the characterization of  $\beta^*$  as:

$$(1 - \alpha^*) \frac{dq^*}{d\beta_i} - \sum_{j=1}^{j=I} (1 - \beta_j^*) \frac{dX_j^*}{d\beta_i} = 0, i = 1, \dots, I. \quad (57)$$

which is the expression given in part (ii) of the PROPOSITION.

Now consider the characterization of the optimal output-share  $\alpha^*$ . Taking the appropriate derivative of (53) yields:

$$\frac{dE[\Pi^L]}{d\alpha} = \frac{dq^*}{d\alpha} - \sum_{i=1}^{i=I} \frac{dX_i^*}{d\alpha} - \omega \frac{de^*}{d\alpha} = 0. \quad (58)$$

Using (55) yields the characterization:

$$(1 - \alpha^*) \frac{dq^*}{d\alpha} - \sum_{i=1}^{i=I} (1 - \beta_i^*) \frac{dX_i^*}{d\alpha} = 0, \quad (59)$$

which is the result presented in part (iv) of the PROPOSITION. Part (v) of the PROPOSITION is immediate by straightforward differentiation of (53) with respect to  $T$ . ■

**Proof.** PROPOSITION 2. Using the same substitutions as in the proof of PROPOSITION 1 yields:

$$E[\Pi^L] = q^* - \sum_{i=1}^{i=I} X_i^* - \omega e^* - \frac{1}{2} \phi \alpha^2 \sigma^2 + \frac{1}{\phi} \ln(-E[U(\Pi^T)]). \quad (60)$$

It is then obvious that the characterization of  $\beta^*$  is the same as that given in PROPOSITION 1, whereas the optimal output-share is obtained by setting  $\frac{dE[\Pi^L]}{d\alpha} = 0$ , which yields:

$$\frac{dq^*}{d\alpha} - \sum_{i=1}^{i=I} \frac{dX_i^*}{d\alpha} - \omega \frac{de^*}{d\alpha} - \phi \alpha^* \sigma^2 = 0. \quad (61)$$

Again exploiting the linear homogeneity property of the restricted profit function, given in (55), one obtains:

$$(1 - \alpha^*) \frac{dq^*}{d\alpha} - \sum_{i=1}^{i=I} (1 - \beta_i^*) \frac{dX_i^*}{d\alpha} - \phi \alpha^* \sigma^2 = 0. \quad (62)$$

■

	All plots			By contract			
	mean	standard deviation		mean			
		total	"within" operator household	owner operator plots	fixed rental plots	share- cropped plots	45 plots
<b>Plot characteristics</b>							
soil type 1 (clay)	0.197	0.398	0.205	0.214	0.160	0.133	
soil type 2 (red earth)	0.182	0.386	0.223	0.134	0.360	0.244	
soil type 3 (sandy)	0.461	0.499	0.268	0.498	0.333	0.400	
soil type 4 (barren)	0.059	0.236	0.173	0.053	0.053	0.111	
irrigated plot dummy	0.892	0.310	0.237	0.901	0.920	0.777	
surface of plot (hectares)	4.372	10.710	7.391	3.457	5.765	8.856	
<b>Crop choice</b>							
wheat	0.200	0.400	0.297	0.194	0.186	0.266	
other grains	0.063	0.244	0.198	0.071	0.040	0.044	
potato	0.096	0.295	0.262	0.095	0.080	0.133	
onions	0.072	0.259	0.229	0.068	0.080	0.088	
garden vegetables	0.235	0.424	0.336	0.262	0.146	0.177	
tomato	0.116	0.321	0.272	0.080	0.266	0.133	
beetroot	0.024	0.153	0.125	0.026	0.000	0.044	
melon	0.076	0.266	0.210	0.053	0.173	0.088	
fodder	0.114	0.318	0.261	0.146	0.026	0.022	
<b>Factor inputs on plot (Tunisian dinars)</b>							
chemical fertilizer	507	1,495	1,268	268	1,125	1,256	
herbicides	230	648	560	127	686	240	
manure	103	538	480	109	104	54	
irrigation	514	1,148	767	365	945	907	
plowing	277	523	397	216	417	495	
seeds	400	1,078	513	347	581	489	
transportation	241	633	494	178	541	208	
harvesting	120	599	491	76	255	231	
<b>Labor inputs on plot (person days)</b>							
female family labor	20	63	39	21	14	28	
female hired labor	151	445	369	80	414	245	
male family labor	86	112	86	72	138	103	
male hired labor	96	239	202	61	206	174	
Output (Tunisian dinars)	6,597	13,890	10,361	4,580	13,861	9,510	

Table 1: Summary statistics for the full sample: plot characteristics, cropping choice, factor inputs and output (455 plots, 96 operator households)



	Mean	Standard deviation	
		total	"within" operator household
Shares accruing to (output) or borne by (costs) the operator (%)			
output	95.31	14.32	8.94
chemical fertilizer	95.86	14.51	8.65
herbicides	95.53	14.97	9.11
manure	96.26	13.97	8.51
irrigation	95.35	16.57	9.68
plowing	94.50	21.34	12.44
family labor	98.77	8.10	6.30
hired labor	97.58	12.62	9.18
seeds	95.85	14.14	8.08
transportation	95.71	16.19	8.47
harvesting	96.48	14.41	8.81
Instrumental variables (landlord characteristics)			
non agricultural income (dinars)	702	1,954	1,177
pension income (dinars)	63	419	128
value of livestock (dinars)	5,475	11,618	5,850
value of agricultural machinery (dinars)	11,074	18,760	10,745
net wealth (dinars)	22,432	38,020	17,377
land ownership (hectares)	22.49	59.39	32.76
schooling of head (years)	4.16	4.57	2.26
age of head (years)	51.83	13.40	6.27
size of household	7.24	4.45	2.15
number of prime-age females	2.06	1.62	0.73
resident landlord dummy (%)	89	30	20
peasant landlord dummy (%)	72	44	27

Table 2: Summary statistics: share of output accruing to, and share of costs borne by the household operating the plot, landlord characteristics (455 plots, 96 operator households)

	Categories of landlords					Categories of tenants		
	pure owner operators	pure landlords	mixed operator landlords	non-resident landlords	non-peasant landlords	pure tenants	mixed owner tenants	77
% of landlords or tenants	64	12	24	8	15	23		
Percentage of plots:								
farmed as owner operator	100	0	41	42	34	0		27
rented out under sharecropping	0	39	20	16	25	40		25
rented out under fixed rental	0	61	39	42	41	60		48
Shares accruing to (output) or borne by (costs) the operator (%) on sharecropped plots								
output	100	51.13	54.04	52.66	50.00	50.00		51.12
chemical fertilizer	100	52.27	63.91	58.33	52.77	50.00		52.18
herbicides	100	52.27	57.39	58.33	52.77	50.00		49.06
manure	100	54.54	69.56	58.33	55.55	50.00		56.25
irrigation	100	47.72	58.08	52.66	55.55	42.30		43.31
plowing	100	43.18	45.60	41.66	52.77	46.15		33.84
family labor	100	79.13	95.65	75.00	80.05	88.46		87.21
hired labor	100	65.90	84.78	58.33	63.88	80.76		75.00
seeds	100	55.27	60.86	58.33	53.66	51.23		53.62
transportation	100	56.81	56.52	58.33	55.55	50.00		48.43
harvesting	100	54.54	73.91	58.33	55.55	57.69		59.37

Table 3: Summary statistics on different categories of landlords and tenants: contractual choice

	Categories of landlords					Categories of tenants		
	pure owner operators	pure landlords	mixed operator landlords	non-resident landlords	non-peasant landlords	pure tenants	mixed owner tenants	77
% of landlords or tenants	64	12	24	8	15	23		
Household characteristics								
non agricultural income (dinars)	733	1,377	139	1,273	1,265	518	700	
pension income (dinars)	38	47	127	0	38	41	84	
value of livestock (dinars)	6,411	1,184	4,135	269	6,363	14,222	7,494	
value of agr. machinery (dinars)	15,076	3,438	8,211	33,375	2,497	10,061	12,777	
net wealth (dinars)	23,344	10,780	28,442	37,004	28,101	24,858	21,204	
land ownership (hectares)	24.08	17.20	52.61	160.38	29.32	4.33	16.99	
schooling of head (years)	4.09	4.71	5.42	12.12	4.36	5.00	4.11	
age of head (years)	52.10	50.80	52.61	47.60	54.38	41.04	46.24	
size of household	7.80	4.63	6.72	4.97	5.26	10.02	9.48	
number of prime-age females	2.30	0.98	1.86	1.35	1.19	2.53	2.85	
resident dummy (%)	97	72	69	0	82	97	96	
peasant dummy (%)	90	21	53	68	0	97	95	

Table 4: Summary statistics on different categories of landlords and tenants: household characteristics (weighted by their relative importance in sample in terms of number of plots)

Method of estimation	Within		Instrumental variables	
	1	2	3	4
Jointly endogenous variable:	sharecropped plot			
Included control variables:				
crop dummies	yes		yes	
crop choice hazard rate		yes		yes
Coefficient associated with sharecropped plot dummy:				
Least squares or 2SLS	-0.487 (-0.90)	-0.262 (-0.47)	-0.156 (-0.22)	0.283 (0.39)
B2SLS (Nagar)			-0.137 (-0.19)	0.315 (0.43)
LIML			-0.110 (-0.15)	0.346 (0.46)
Reverse B2SLS (Nagar)			-44.042 (-0.97)	15.700 (2.23)
Fuller			-0.113 (-0.15)	0.343 (0.45)
$R^2$	0.499	0.442		
Test of OID restrictions: $p$ -value				
2SLS-based			0.032	0.074
LIML/Fuller-based			0.027	0.068
Weak IV null on sharecropped plot reduced form:				
partial $F$			20.404	21.262
partial $R^2$			0.336	0.345
Hahn-Hausman (2002) IV validity test:				
$m_2 \sim t$ -statistic [ $p$ -value]			2.107 [0.035]	-2.081 [0.038]

Table 5: The impact of share tenancy on log output per hectare (mean = 6.703, std. dev.= 2.302): operator household-specific effects in all specifications; four soil type dummies (mixed soil type is excluded category), irrigated plot dummy, and log plot area included in all specifications (455 plots, t-statistics in parentheses)

Method of estimation	Instrumental variables		
	1	2	3
Jointly endogenous variables:			
sharecropped plot	yes	yes	yes
plot area	yes	yes	
plot rented out	yes		
crop choice hazard rate	yes	yes	yes
Coefficient associated with sharecropped plot dummy:			
2SLS	-0.327 (-0.23)	-0.811 (-0.58)	-0.548 (-0.41)
B2SLS (Nagar)	-1.990 (-0.98)	-2.360 (-1.12)	-1.760 (-0.92)
LIML	-1.624 (-0.75)	-2.150 (-0.96)	-1.734 (-0.82)
Reverse B2SLS (Nagar)	0.333 (2.74)	-1.032 (-2.57)	-1.540 (-2.51)
Fuller	-1.312 (-0.67)	-1.857 (-0.91)	-1.474 (-0.77)
Test of OID restrictions: $p$ -value			
2SLS-based	0.201	0.210	0.240
LIML/Fuller-based	0.347	0.418	0.468
Weak IV null on sharecropped plot reduced form:			
partial $F$	25.972	10.996	10.659
partial $R^2$	0.392	0.214	0.209
Hahn-Hausman (2002) IV validity test:			
$m_3 \sim t$ -statistic [ $p$ -value]	-0.206 [0.836]	-0.090 [0.927]	-0.017 [0.985]
Andrews (1999) IV selection criteria			
IV-BIC	-38.197	-43.207	-48.490
IV-AIC	-8.227	-9.491	-11.028
IV-HQ	-20.344	-23.122	-26.174

Table 6: The impact of share tenancy on log output per hectare (mean = 6.703, std. dev. = 2.302): operator household-specific effects in all specifications; four soil type dummies (mixed soil type is excluded category), irrigated plot dummy, log plot area, rented out dummy, and crop choice hazard rate included in all specifications (455 plots, t-statistics in parentheses)

	Joint significance of $\beta_i/\alpha$ variables $\chi^2_{10}$ - statistic [ <i>p</i> -value]		Test of OID restrictions $F$ - statistic [ <i>p</i> -value]		Hahn-Hausman IV validity test $m_3$ - statistic [ <i>p</i> -value]
	2SLS	Fuller	2SLS	Fuller	
Output	10.942 [0.362]	11.876 [0.293]	0.958 [0.328]	0.326 [0.568]	0.626 [0.531]
Ch. fert.	6.542 [0.767]	9.382 [0.496]	0.299 [0.584]	0.202 [0.653]	9.198 [0.000]
Herb.	10.154 [0.427]	10.868 [0.367]	2.045 [0.153]	0.334 [0.563]	0.421 [0.673]
Manure	4.805 [0.903]	5.613 [0.846]	0.015 [0.902]	0.015 [0.902]	7.370 [0.000]
Irrigation	16.466 [0.087]	18.741 [0.043]	3.731 [0.054]	0.335 [0.563]	0.129 [0.896]
Plowing	5.525 [0.853]	11.651 [0.309]	0.154 [0.695]	0.118 [0.731]	-2.744 [0.006]
Fam. lab.	4.431 [0.925]	6.990 [0.726]	2.394 [0.123]	0.318 [0.573]	0.972 [0.331]
Hire lab.	3.503 [0.966]	10.559 [0.392]	0.179 [0.672]	0.125 [0.723]	6.371 [0.000]
Seeds	8.072 [0.621]	9.483 [0.486]	0.687 [0.408]	0.308 [0.579]	-6.789 [0.000]
Trans.	14.669 [0.144]	18.691 [0.044]	0.638 [0.425]	0.288 [0.591]	3.609 [0.000]
Harvest.	3.618 [0.962]	8.011 [0.627]	0.241 [0.623]	0.158 [0.691]	-2.103 [0.036]

Table 7: The impact of share tenancy on log output and log input use per hectare: 10 cost-to-output share ratios included in each specification, operator household-specific effects in all specifications; crop choice hazard rate and cost-to-output share ratios assumed jointly endogenous (455 plots)

	Output	Chem fert.	Herb.	Manu.	Irrig.	Plow.	Fam. lab.	Hired. lab.	Seed	Trans.	Harv.
$\delta_i$ coeffs	-0.548 (-2.39)	-0.305 (-2.08)		0.032 (2.00)	0.042 (2.54)		0.726 (2.84)	-0.341 (-2.75)		0.271 (3.08)	-0.044 (-2.32)
$\gamma_{ij}$ coeffs											
Output	1.696 (3.00)										
Ch. fert.	-0.424 (-2.12)										
Herb.	-0.138 (-2.76)	-0.038 (-2.92)									
Manure	0.017 (2.30)	-0.029 (-1.98)	-0.046 (-2.88)	-0.018 (-2.57)							
Irrigation	-0.067 (-2.35)		0.036 (2.91)	0.009 (2.35)	0.073 (2.82)						
Plowing	0.003 (2.18)		-0.012 (-2.93)	-0.001 (-2.75)		-0.041 (-3.11)					
Fam. lab.			-0.026 (-2.25)	-0.015 (-2.07)		-0.263 (-3.11)					
Hire lab.	-1.242 (-3.03)		0.062 (2.76)	0.055 (2.42)			0.051 (2.17)				
Seeds	-0.176 (-2.31)	0.713 (2.39)		0.059 (2.49)		0.256 (3.14)	-0.373 (-2.02)	1.306 (3.03)	-1.507 (-3.11)		
Trans.			0.038 (2.93)	0.005 (2.63)		0.034 (2.51)	0.307 (2.49)	-0.208 (-2.09)	-0.142 (-2.02)		
Harvest.	-0.014 (-2.47)		-0.031 (-2.52)	-0.046 (-2.27)							
$R^2$	0.088	0.060	0.063	0.057	0.362	0.117	0.112	0.128	0.054	0.053	0.155
Test of optimality of cost-sharing as given in Propositions 1(i) and 2(i), and empirical counterpart given by equation (51) in text [ $p$ -value]											
		-0.117 [0.109]	0.033 [0.024]	-0.047 [0.053]	-0.466 [0.000]	0.080 [0.336]	-0.499 [0.007]	0.026 [0.557]	-0.875 [0.006]	-0.221 [0.000]	0.039 [0.423]

Table 8: The translog restricted profit function representation of the production technology. Homogeneity and symmetry imposed; cost-shares and cropping hazard rate assumed exogenous; non-linear household-specific effects included (455 plots, Huber-White t-statistics in parentheses, p-values in square brackets, only coefficients significant at better than the 5 percent level are reported)

	Output	Chem fert.	Herb.	Manu.	Irrig.	Plow.	Fam. lab.	Hired. lab.	Seed	Trans.	Harv.
$\delta_i$ coeffs	-0.337 (-2.05)	-0.674 (-3.79)					1.083 (5.12)	-0.752 (-5.23)	0.515 (3.60)	0.331 (4.70)	-0.127 (-2.60)
$\gamma_{ij}$ coeffs											
Output	1.910 (4.74)										
Ch. fert.											
Herb.			0.471 (4.20)								
Manure	0.043 (2.89)	-0.083 (-2.67)									
Irrigation	-0.142 (-2.65)		0.055 (4.91)	0.011 (4.61)	0.129 (5.70)						
Plowing		0.180 (3.55)	-0.041 (-3.13)	-0.005 (-2.60)	0.062 (3.88)	-0.108 (-6.34)					
Fam. lab.		0.513 (2.15)	-0.203 (-2.92)			-0.657 (-7.30)					
Hire lab.	-2.641 (-7.01)	-0.217 (-2.57)	0.161 (5.11)	0.068 (4.44)	0.036 (5.20)	0.036 (5.20)	0.108 (2.06)	-0.084 (-4.10)			
Seeds	0.674 (4.49)	0.375 (2.08)	-0.322 (-3.01)	0.039 (2.36)	0.443 (7.01)	0.443 (7.01)	-0.954 (-4.08)	2.774 (7.04)	-2.907 (-7.12)		
Trans.			0.061 (3.21)	0.008 (2.40)	-0.026 (-2.50)	0.068 (5.64)	0.532 (6.11)	-0.294 (-4.86)	-0.242 (-4.89)		
Harvest.	-0.029 (-2.33)	-0.111 (-3.77)	-0.074 (-4.46)	-0.053 (-4.47)	0.015 (4.53)		0.064 (3.03)	0.070 (5.01)	0.148 (5.64)	0.010 (4.14)	-0.044 (-5.03)
Test of optimality of cost-sharing as given in Propositions 1(i) and 2(i), and empirical counterpart given by equation (51) in text [ <i>p</i> -value]											
	-0.559 [0.009]	0.281 [0.000]	-0.034 [0.003]	-0.757 [0.000]	0.022 [0.795]	-1.599 [0.000]	-0.021 [0.648]	-1.739 [0.000]	-0.561 [0.000]	-0.036 [0.057]	

Table 9: The translog restricted profit function representation of the production technology. Homogeneity and symmetry imposed; output-share, cost-shares and cropping hazard rate assumed jointly endogenous; non-linear household-specific effects included (455 plots, Huber-White t-statistics in parentheses, p-values in square brackets, only coefficients significant at better than the 5 percent level are reported)