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SURVIVAL CONVERGENCE: SPECIFICATION MATTERS

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Abstract

Retaining as a "Millennium Development Goal" a decrease by two thirds in child mortality whatever its initial level assumes that the target elasticity of child mortality may be the same in all the developing countries. We show that such an assumption is not perfectly consistent with the bounded characteristic of the child mortality indicator. We propose to use the logit transformation of such bounded human development indicators to have appropriate estimates of human convergence: this provides the best specification of the overall change in child survival among countries. Applied to child survival on a cross-section basis for 166 countries and over a thirty-year period (1970-99), this measure shows no evidence of absolute convergence but on the contrary a highly significant divergence.

JEL classification: I12, O11, C31.

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1. Introduction

Child survival is often considered as one of the most relevant gross indicators of "human development" (see for instance the Sachs Report of the Commission on Macroeconomics and Health, WHO 2001). Indeed, the two-third reduction in child mortality is one of the Millennium Development Goals as defined by the United Nations, while Sen has suggested mortality to be *"an indicator of economic policy success or failure"* (Sen 1998). It is henceforth essential to assess consistently the speed to which the child survival rate, i.e. the proportion of children dying before the age of five changes.

The analysis of the movements in child survival gaps can be carried out similarly to that of income gaps. The assumption convergence states that relative gaps tend to decrease over time. However, while the empirical literature on convergence is very abundant with regard to economic growth, very few econometric studies (Ram 1998, Sab and Smith 2002, Hobijn and Franses 2002, Mazumdar 2003, Neumayer 2004) focus on survival convergence.

The study of human development indicators, and notably of the increase in child survival, is however different from that of growth aggregates such as industrial production, investment or trade. The movements in these indicators being bounded, analyzing "human development convergence" raises specific methodological issues, which may bias the estimates towards "convergence" if unsolved. As human development indicators have an upper bound, the relative position of a given country (towards this upper bound) has to be taken into account (see Kakwani 1993, Anand and

Ravallion 1993). Thus, two reasons make this analysis different from that of economic growth: (1) human development variables such as mortality or survival have an upper bound and (2) this bound is the same for all the countries. Omitting one of these two features when studying convergence/divergence would lead to a bias in the convergence estimates. According to the neo-classical analysis of economic growth, the catching up occurs spontaneously due to technical progress diffusion as well as to capital accumulation. In the case of human development, convergence could only result from the (bounded) shape of the child survival variable.

This paper aims at explaining the reasons why the several measures of human development used in the empirical literature can lead to some contradictory findings with regard to convergence. We rely in particular on the most often used specifications in the literature on child survival convergence. Each of these specifications is discussed and it is evidenced that the most appropriate way to assess "human convergence" is to use the logit transformation of the considered human development indicator (child survival in this paper), weighting the actual level of survival by the distance to its upper bound. Last, we not only test convergence, but also (and in particular) the specification used in each case, since a mis-specification can lead to either the illusion or to the overestimation of convergence.

Section 2 discusses the estimates of the child survival dynamic in the literature as well as the implicit assumptions associated to the estimated impacts. Section 3 presents the econometric issues and draws the tests to be applied to the previously analyzed functional forms. Section 4 presents the results. Section 5 concludes.

2. Conflicting measures of human development, illustrated by under-five survival.

When measuring "human development", much attention have been paid to the choice and to the aggregation of indicators, a choice inevitably questionable.[1] A preliminary question is to design a relevant measure so that it can be compared across countries or over time. We argue that the answer depends on the aim of the index. If health status is designed as a human capital indicator, i.e. an input indicator source of economic growth, it has to reflect the ability to generate income. A decreasing marginal productivity of the considered human factor is then to be assumed (i.e. the first derivative of the health production towards human factors has to be positive and the second derivative negative as illustrated in figure 1). If, on the contrary, as in Grossman (1999), health is desired *per se*, produced with a decreasing marginal productivity, the output indicator has to reflect the ability to increase the resource itself and has to be considered as a health performance index. A same absolute increase indicates an all the higher performance as the initial level was already high (i.e. the first and the second derivative of the indicator have to be positive as illustrated in the figure 1).

Insert Figure 1 here

It is obviously all the more difficult to improve a human development indicator, as it gets closer to its upper limit. Table 1 most simply illustrates the different ways to measure a change in life expectancy, according to the specification used.

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[1]Cf. the extensive literature since the publication of the *Human Development Report* (1989) or the works of the CDP (*United Nations Committee for Development Policy*) about the APQLI (Augmented Quality of Life Index), which became the HAI (Human Assets Index) in 2002.

Insert Table 1 here

As a result, the concept of convergence applied to human development (for our purpose, to survival) can be expressed in different and possibly conflicting ways, which fundamentally differentiates economic convergence, i.e. incomes convergence, from health or education convergence. We successively consider the two traditional concepts of convergence, the sigma and the beta convergence, applying them to human development. This will enable us to compare the several measures used in the empirical literature.

Sigma-convergence in human development

First, the sigma-convergence is to be considered, i.e. the move in the dispersion (the standard deviation) of the indicator (in this study the survival rate). In the literature on the macroeconomic factors of health (see for instance Anand and Ravallion 1993, bidani and Ravallion 1997, Filmer and Pritchett 1997, Filmer and Hammer 2000, Kenny 2005), three indices are mostly used, leading to three alternative measures of σ-convergence[2].

- The absolute value of the indicator: σ-convergence here means a decrease in the absolute differences of the survival rates;
- The logarithmic value of the indicator: σ-convergence here means a decrease in the relative differences;
- The logarithm of the distance between the actual level of the indicator and its upper limit: σ-convergence here means a decrease in the relative differences of

[2] Other measures of the dispersion are sometimes used. Bourguignon and Morrisson (2002) rely on Theil index applied to life expectancy as well as on the variance of the life expectancy variance. Cornia and Menchini (2005) apply Gini index to infant mortality rates as well as to the distance to the maximum level of life expectancy attainable (100 – Life Expectancy).

the distance to the maximum level[3]. This third measure has been in particular used by Anand and Ravallion (1993) or Hobijn and Franses (2001).

A fourth measure of the indicator, initially proposed by Bhalla and Glewwe (1986), has been quite rarely used.

- The logarithm of the actual level of the indicator related to the distance between this level and the maximum level it can reach.

In the context of the Millennium Development Goals defined by the United Nations, one can ask whether the fourth millennium goal (lowering the child mortality by two thirds) implies sigma-convergence or not for the countries reaching that target. The answer depends on the indicator used. Achieving a two thirds reduction in child mortality means a sigma-convergence in absolute terms or in logs of child survival, while it means no convergence (nor divergence) in the logs of child mortality. Suppose for instance two countries with respective child mortality rates of 300 and 30 per thousand (survival of 700 and 970), i.e. an absolute gap of 270. Reducing the child mortality by two thirds in both countries leads to new level of 100 and 10 (900 and 990), i.e. an absolute gap of 90 (instead of 270), a smaller relative gap of survival rates (990/900=1.1 instead of 970/700=1.4) and an unchanged relative gap of mortality rates (ten to one).

Figures 2.1 to 2.4 illustrate the potentially conflicting results, for under-five survival sigma-convergence, according to the definition we chose. They deal with 166 countries (cf. Appendix 5) for seven 5-year periods covering 1965-1999. First, the

[3] If the indicator is child survival and if the upper bound is 1, the sigma-convergence corresponds to a decrease in the child mortality relative differences.

absolute (as well as logarithmic) levels of survival are "converging": relative as well as absolute differences between the rates of survival have been decreased over time. Second, the relative differences between the distance from the actual to the maximum levels, i.e. the relative differences between the child mortality rates, are diverging, as well as the relative differences between child survival weighted by the distance to the upper bound.

Insert Figure 2 here

Beta-convergence in human development

Sigma-convergence is just a description, without any assumption on the functional relation, contrary to the beta-convergence. The beta-convergence corresponds to the hypothesis that the growth rate of the interest variable (usually the per capita income, here the human development and more precisely child survival) depends negatively on its prior value, due to decreasing returns. Therefore to be tested is the relation linking the child survival growth rate (the difference of the logs) and the child survival initial level (in logarithms), controlling or not for the influence of other factors, which respectively corresponds to "absolute" or "conditional" convergence. We may assume that the principle of decreasing returns fits for the production of health as well as for the production of goods.

Combining the literature on income convergence and the studies on the determinants of mortality or survival, several recent studies examine specifically the "convergence" between countries with regard to life expectancy (or to some other indicators of human development), or simply consider factors explaining rates of human

indicators change among countries, controlling for the prior level of the indicator (which involves an implicit test of human convergence). Table 2 presents a summary of such studies applied to mortality or survival.

Insert Table 2 here

Ram (1998) assesses the variation of life expectancy, either in absolute value or in logarithm, depending on its initial value and its initial value squared, adding the per capita income as an explanatory variable (conditional convergence). With relative or absolute difference as well (i.e variables transformed in logarithm or not) and from a sample of 123 countries over the 40-year period 1950-90, he finds convergence at an accelerating rate from a certain threshold of the initial level equals to 35 years, i.e. for the countries with an initial level of life expectancy superior to 35 years, which covers fairly the whole sample. Sab and Smith (2002) test a similar relationship (in log) for both the variation of life expectancy and that of the literacy rate, respectively on the initial level and the squared initial level as explanatory variables, controlling for the evolution of each variable by the prior level of the other variable, in a simultaneous equations system: they conclude to (conditional) convergence for each variable (on 100 countries and on the 20-year period 1975-96)[4]. Neumayer (2004) studies the impact of AIDS on life expectancy or child survival convergence/divergence (in cross-section, for 186 countries, for five 10-year different periods). While relying mostly on sigma-convergence, Neumayer evidences from a log-log specification an all the stronger convergence as AIDS prevalence is low.

[4] A similar conclusion for convergence on education is obtained by Zhang and Li (2002).

The functional forms used in these studies can be debated when studying convergence, since they deal with bounded variables, as human development indicators are. As the convergence results are to some extent pre-determined when related to the relative change of a variable with an upper bound, the conclusions on convergence in the previous studies are not surprising. Indeed, the functional forms they use are not appropriate to the study of survival function. That is why, as presented in the table 3, Anand and Ravallion (1993) or Hobijn and Franses (2002) transform the explained variable (survival) in order to reflect an "achievement", namely the log of the distance between the maximum level attainable (of survival) and its actual level. They conclude to divergence (on 150 countries for the 35-year period 1965-1990) when they consider the achievement indicator, to convergence when the explained variable is the absolute level of the indicator.

Insert Table 3 here

An appropriate specification of human convergence

We only consider absolute convergence, the relationship between the current level and the initial level of survival being tested, without controlling for the influence of other factors. Moreover, we only consider child survival as human development indicator.

Following the literature previously reviewed, four basic specifications are identified to study child survival absolute convergence, $s_{i,t}$ being the rate of child survival of the country i, $s_{i,t0}$ the initial rate of child survival of the country i

$$s_{i,t} = \alpha_1 + \beta_1 . s_{i,t0} + \varepsilon_i \quad (1)$$

$$\ln s_{i,t} = \alpha_2 + \beta_2.\ln s_{i,t0} + e_i \quad (2)$$

$$\ln s_{i,t} = \alpha_3 + \beta_3.\ln s_{i,t0} + \gamma_3.(\ln s_{i,t0})^2 + e_i \quad (3)$$

$$-\ln(1-s_{i,t}) = \alpha_4 + \beta_4.(-1).\ln(1-s_{i,t0}) + u_i \quad (4)$$

$$\ln \frac{s_{i,t}}{1-s_{i,t}} = \alpha_5 + \beta_5.\ln \frac{s_{i,t-1}}{1-s_{i,t0}} + \xi_i \quad (5)$$

The first three measures can be used for income per capita or for human development as well. The fourth one is, on the contrary, specific to any indicator of human development with an upper bound (such as life expectancy, under-five survival, enrollment ratios, etc.). A fifth specification is added to these four ones, from a measure used by Bhalla and Glewwe (1986), which is appropriate for an indicator with an upper bound. It is the logit transformation of the survival rate, i.e. the log of the ratio of the survival indicator (for their purpose life expectancy) to the difference between the maximum value and the actual value of this indicator (cf. Table 2), which is a specific measure of human development. This measure is appropriate for human development indicators with an upper bound, as the fourth one.

These five specifications rely respectively on the following parameters, supposed to reflect a "natural path" of the child survival variable;

$$\frac{ds_{i,t}}{ds_{i,t0}} = \beta_1 \quad (6)$$

$$\frac{ds_{i,t} / s_{i,t}}{ds_{i,t0} / s_{i,t0}} = \beta_2 \quad (7)$$

$$\frac{ds_{i,t} / s_{i,t}}{ds_{i,t0} / s_{i,t0}} = \beta_3 + \gamma_3.\ln s_{i,t0} \quad (8)$$

$$\frac{ds_{i,t}}{ds_{i,t0}} = \beta_4.\left(\frac{1-s_{i,t}}{1-s_{i,t0}}\right) \quad (9)$$

$$\frac{ds_{i,t} / s_{i,t}}{ds_{i,t0} / s_{i,t0}} = \beta_5.(\frac{1-s_{i,t}}{1-s_{i,t0}}) = \beta'_5 \quad (10)$$

The first two measures assume either a constant marginal impact (6) or a constant elasticity (7) whatever the initial level is. The reason why the first two specifications are not appropriate to the study of convergence is that they impose a constant impact, what is not consistent with the existence of an upper bound to the explained variable. This shortcoming does not appear with the last three others. The third specification implicitly assumes an elasticity that moves along with the survival rate and the sign of which may change beyond a certain threshold (if $\beta_3>0$ and $\gamma_3<0$ in equation 8). The third one is still debatable, since according to the location of that threshold, the marginal impact of the previous level on the actual level could possibly be zero or even be negative.

Both the fourth (the log of the difference between the upper bound and the actual level of the indicator) and the fifth formulation (logit transformation of survival) have their first and second derivative positive[5]. However, two argues make the fifth one more suitable. First, the logit transformation (5) provides elasticity instead of marginal impact, (as with equation 4), which facilitates the interpretations. The second, and somewhat more important concern deals with the interpretation to give to the impact from (9) or (10). Indeed, from the fourth specification:

Given $s_{i,t} = 1 - m_{i,t}$ (11)

We have $ds_i = s_{i,t} - s_{i,t0} = - m_{i,t} + m_{i,t0} = - dm_i$ (12)

Hence $$\frac{ds_{i,t}}{ds_{i,t0}} = -\frac{dm_{i,t}}{dm_{i,t0}} = \beta'_4 = \beta_4.(\frac{m_{i,t}}{m_{i,t0}}),$$ (13)

[5] Actually, the second derivative is only positive from a survival rate superior to the half of its maximum level, i.e. 500 per thousand, which corresponds to the entire sample.

And $$\frac{ds_{i,t}/(1-s_{i,t})}{ds_{i,t0}/(1-s_{i,t0})} = -\frac{dm_{i,t}/m_{i,t}}{dm_{i,t0}/m_{i,t0}} = \beta_4 \quad (14)$$

Therefore, using the fourth specification leads to assume a constant elasticity along with the decrease in child mortality. The last formulation (5) escapes the previous criticism. As a matter of fact, it explicitly takes into account the fact that the variable is bounded: a same marginal variation is an all the higher performance, as we get closer to the upper bound. The estimated impact is $\beta_5 = \frac{ds_{i,t}/s_{i,t}}{ds_{i,t0}/s_{i,t0}} .(\frac{1-s_{i,t0}}{1-s_{i,t}})$, the elasticity of $s_{i,t}$ to $s_{i,t0}$ being weighted by the ratio of the distance to the upper bound in t and in $t_{0;}$

- When $s_{i,t} < s_{i,t0}$ (decreasing survival), then $\beta_5 < \frac{ds_{i,t}/s_{i,t}}{ds_{i,t0}/s_{i,t0}}$,
- when $s_{i,t} > s_{i,t0}$ (increasing survival), then then $\beta_5 > \frac{ds_{i,t}/s_{i,t}}{ds_{i,t0}/s_{i,t0}}$,

Consistently with a measure of the performance, the impact from the last formulation is all the higher as survival gets closer to its upper bound.

These measures of absolute convergence are now to be tested, keeping in mind a doublet target: first testing empirically the previous specifications used in the literature, and second comparing their results with regard to convergence/divergence.

3. Econometric issues: choosing an appropriate approach

Cross-section rather than panel data

The econometric study relies on a cross-section analysis, for two reasons that make it more suitable for our purpose than a panel analysis, even if the data we have would have enabled us to test a dynamic panel specification with the usual advantages of panel data.

Firstly, absolute convergence in human development is a long-term process. Assessing such a long-term process from a dynamic panel relying on 5-year periods (for instance) would then be less appropriate than a cross-section analysis over a 30-year period.

Secondly, the country specific effects used to control for the unobservable heterogeneity would lead to test conditional convergence instead of absolute convergence, which is our first aim.

The specification tests

The estimates rely on a sample of 166 countries over the 30-year period 1970-1999. The data come from Ahmad, Lopez and Inoue, (2000)[6]. The specifications previously detailed are explicitly tested. We not only use the test from Godfrey-Wickens (1981) that relies on the box-cox transformation but also the Ramsey-Reset (1969) to check the robustness of the results. It is thereafter evidenced that the use of a good specification leads to significantly downwards the level of convergence firstly assessed, or can even lead to conclude on divergence. Two levels of test are henceforth provided: first, the hypothesis of good specification, then the hypothesis of absolute convergence, according to the functional form.

The Godfrey-Wickens' test relies on the Box and Cox transformation; $F(x,\lambda)=\frac{x^{\lambda}-1}{\lambda}$. Depending on the value of λ, any linear or non-linear specification can be tested. Setting $\lambda = 1$ the null hypothesis is a linear specification, providing there is a constant in the regression. To test a log-linear specification we set $\lambda = 0$, which leads to

[6] See Appendix 1 for detailed comments on those data.

consider a logarithmic transformation of the variable, $F(x, 0) = lnx$. To test a quadratic specification, we set $\lambda = 2$, leading to consider the expression $F(x,2) = \frac{x^2 - 1}{2}$. Last, if the null is a logit specification, we set $\lambda = 0$, and operate a change in the variable x to $y = \frac{x}{1-\lambda}$. In all the cases, the alternative assumption, H_1, captures any other functional form.

- Linear specification: $\lambda = 1$

$$H_0 : F(s_{i,t}, 1) = \alpha + \beta.F(s_{i,t0}, 1) + \varepsilon_i \quad (15)$$

$$H_1 : F(s_{i,t}, \lambda) = \alpha' + \beta'.F(s_{i,t0}, \lambda) + \nu_i \quad (16)$$

We replace $F(s_{i,t}, 1)$ by s_i and $F(s_i, \lambda)$ by its limited development around 1;

$$H_0 : s_{i,t} = \alpha + \beta.s_{i,t0} + \varepsilon_i \quad (17)$$

$$H_1: \frac{s_{i,t} - 1}{\lambda} + \ln(s_{i,t}).(\frac{\lambda - 1}{\lambda}).s_{i,t} = \alpha' + \beta'(\frac{s_{i,t0} - 1}{\lambda}) + \ln(s_{i,t0}).(\frac{\lambda - 1}{\lambda}).s_{i,t0} + \nu_i \quad (18)$$

Which leads to test econometrically;

$$H_0: s_{i,t} = \alpha + \beta.s_{i,t0} + \varepsilon_i \quad (19)$$

$$H_1: s_{i,t} = \alpha' + \beta'.s_{i,t0} + (\lambda-1).[\ \hat{\beta}\ .s_{i,t0}.\ln(s_{it0}) - \hat{s}_{i,t}.\ln(\hat{s}_{i,t})\ ] + \nu_i \quad (20)$$

If we call $q_1 = \hat{\beta}\ .s_{i,t0}.\ln(s_{it0}) - \hat{s}_{i,t}.\ln(\hat{s}_{i,t})$, then we test

$$H_0: s_{i,t} = \alpha + \beta.s_{i,t0} + \varepsilon_i \quad (21)$$

$$H_1: s_{i,t} = \alpha' + \beta'.s_{i,t0} + q_1.(\lambda-1) + \nu_i \quad (22)$$

The test variable is q_1. To avoid any simultaneous bias, the values of $lns_{i,t}$ $s_{i,t}$ and β in the expression of q_1 (under brackets equation 20) must be those predicted under H_0. Hence, if the coefficient associated to q_1 is not significantly different from 0 (i.e. λ is not significantly different from 1), the null hypothesis cannot be rejected and we

conclude that the linear formulation is the best one. Otherwise, the linear specification is rejected.

- Log-linear specification: $\lambda = 0$

$$H_0: F(s_{i,t}, 0) = \alpha + \beta.F(s_{i,t0}, 0) + \varepsilon_i \quad (23)$$

$$H_1: F(s_{i,t}, \lambda) = \alpha' + \beta'.F(s_{i,t0}, \lambda) + \nu_i \quad (24)$$

Replacing $F(s_{i,t}, 0)$ by $\ln(s_{i,t})$ and $F(s_{i,t}, \lambda)$ by its limited development around 0;

$$H_0: lns_{i,t} = \alpha + \beta.lns_{i,t0} + \varepsilon_i \quad (25)$$

$$H_1: lns_{i,t} + \frac{\lambda}{2}.(lns_{i,t})^2 = \alpha' + \beta'.lns_{i,t0} + \frac{\lambda}{2}.(lns_{i,t0})^2 + \nu_i \quad (26)$$

Which leads to test econometrically;

$$H_0: lns_{i,t} = \alpha + \beta.lns_{i,t0} + \varepsilon_i \quad (27)$$

$$H_1: lns_{i,t} = \alpha' + \beta'.lns_{i,t0} + \frac{\lambda}{2}.[-(\ln s_{i,t})^2 + \hat{\beta}.(lns_{i,t0})^2] + \nu_i \quad (28)$$

We call $q_2 = \frac{1}{2}.[-(\ln s_{i,t})^2 + \hat{\beta}.(lns_{i,t0})^2]$,

$$H_0: lns_{i,t} = \alpha + \beta.lns_{i,t0} + \varepsilon_i \quad (29)$$

$$H_1: lns_{i,t} = \alpha' + \beta'.lns_{i,t0} + \lambda.q_2 + \nu_i \quad (30)$$

q_2 is the test variable. To avoid any simultaneous bias, the value of $lns_{i,t}$ and of β in the expression of q_2 (under brackets equation 28) must be those predicted under H_0. Hence, if the coefficient associated to q_2 is not significantly different from 0, the null hypothesis is not rejected and we conclude that the log-linear formulation is the most suitable for our purpose. Otherwise, the log-linear specification is rejected.

- Quadratic specification: $\lambda = 2$

We want to test a logarithmic-quadratic functional form, i.e;

$$lns_{i,t} = \alpha + \beta.lns_{i,t0} + \gamma.(lns_{i,t0})^2 + \varepsilon_i$$

Then, we pose;

$$H_0: \ln s_{i,t} = \alpha + \beta.\ln s_{i,t0} + \gamma.\, F(\ln s_{i,t0}, 2) + \varepsilon_i \quad (31)$$

$$H_1: \ln s_{i,t} = \alpha' + \beta'.\ln s_{i,t0} + \gamma'.\, F(\ln s_{i,t0}, \lambda) + \nu_i \quad (32)$$

Replacing $F(\ln s_{i,t}, 2)$ by $(\ln s_{i,t0})^2$ and $F(s_{i,t}, \lambda)$ by its limited development around 2 :

$$H_0: \ln s_{i,t} = \alpha + \beta.\ln s_{i,t0} + \gamma.(\ln s_{i,t0})^2 + \varepsilon_i \quad (33)$$

$$H_1: \ln s_{i,t} = \alpha' + \beta'.\ln s_{i,t0} + \gamma'.\left[\frac{(\ln s_{i,t0})^2 - 1}{\lambda} + \frac{\lambda - 2}{\lambda}.\ln(\ln s_{i,t0}).(\ln s_{i,t0})^2\right] + \nu_i \quad (34)$$

when $\lambda=2$, the functional form is quadratic.

From H_0, we estimated γ and define q_3, the following test variable:

$$q_3 = \hat{\gamma}_{H0}.\ln(\ln s_{i,t0}).(\ln s_{i,t0})^2,$$

which leads to test econometrically:

$$H_0 : \ln s_{i,t} = \alpha + \beta.\ln s_{i,t0} + \gamma.(\ln s_{i,t0})^2 + \varepsilon_i \quad (35)$$

$$H_1 : \ln s_{i,t} = \alpha + \beta.\ln s_{i,t0} + \gamma.(\ln s_{i,t0})^2 + \delta.q_3 + \nu_i \quad (36)$$

Hence, if the coefficient associated to q_3 (γ) is not significantly different from 0, the null hypothesis is not rejected and we conclude that the quadratic log-linear formulation is the most suitable for our purpose. Otherwise, that functional form is rejected.

- Logit specification

To test the consistency of the logit formulation, i.e. $\ln(\frac{s_{i,t}}{1-s_{i,t}})$, we set λ=0 and change the variable by defining $s_{it}^{*} = (\frac{s_{it}}{1-s_{it}})$. Then, we come back to the equations (23) to (30) with respectively s_{it}^{*} and $s_{i,t0}^{*}$ instead of $s_{i,t}$ and $s_{i,t0}$.

The Ramsey-Reset test consists on a joint significance test of the coefficients associated to the predicted value of the dependant variable raised at the power 2, 3 and 4 and simultaneously included in the initial regression. If the three coefficients are jointly not significantly different from 0, the null hypothesis of good specification is not rejected.

4. Results

The different hypotheses are tested using the Generalized Least Squares estimator with White correction to have robust error-standards. Table 4 reports the results of the GLS estimates, as well as of the Godfrey-Wickens test and of the Ramsey-Reset test[7]. The dependent variable relies on average data over 1970-99, the RHS variable on average data over 1965-69[8]. Appendix 1 provides detailed information on the origin of the data.

[7] Since we estimate $s_{i,t} = \alpha + \beta.s_{i,t-1}$, there is convergence if β<1. Hence we test β=1 versus β<1, that is why we indicate the standard-errors associated to the coefficients.

[8] To check the robustness of our results, we have also tested these specifications with the dependent variable at the end of the period (1995-1999) and the delayed variable at the beginning of the period (1970-1974). The findings remain qualitatively unchanged, with regard to the rejection or not of

Insert Table 4 here

The estimates strongly confirm our hypotheses.

First, both the linear specification (column 1 of table 4) and the log-linear one (column 2 of table 4) wrongly conclude to a bêta-convergence phenomenon. Indeed, the coefficients associated to the delayed variable are significantly inferior to 1, while the Godfrey-Wickens test rejects both the linear and the log-linear specification, with p-values, i.e. probabilities of committing a Type 1 error, respectively inferior to 1% and 5%. The more general Ramsey-Reset test strongly confirms these results (with p-values respectively equal to 0.0001 and 0.0002). This validates our assumption that these specifications wrongly lead to conclude on convergence by constraining either the marginal impact or the elasticity to be constant, whatever the initial level.

Second, the quadratic-logarithmic function used by Ram (1998) is also rejected by both the Godfrey-Wickens and the Ramsey-Reset test with a Type 1 error inferior to 1% for each of these tests (column 3 of table 4). Note that with that specification, neither the coefficient associated to the delayed variable nor the one associated to the quadratic term is significantly different from 0. The rejection of that functional form may be due to our concern on the implicit assumption of a reversal threshold tested with that formulation.

the functional form (Godfrey-Wickens or Ramsey-Reset tests) as well as to the convergence/divergence results.

The specification previously used by Hobijn and Franses (2001), which takes into account the distance to the upper bound does not conclude to convergence (hence the problem of the bounded indicator may have been solved), but this specification is rejected by both the Godfrey-Wickens and the Ramsey-Reset tests (column 4 in table 4), which confirms our suspicion on the constant elasticity it imposes along with the decrease in child mortality.

Last, the null hypothesis of good specification of the logit specification (column 5 in table 4) is neither rejected by the Godfrey-Wickens nor by the Ramsey-Reset with Type 1 errors respectively equals to 89% and 53%. Moreover, the hypothesis of convergence is widely rejected. In other words, child survival, measured by its logit transformation, tends to be all the more improved as it is initially higher.

5. Conclusion

The fourth Millenium Development Goal implicitly raises the question of a same "target elasticity" for the decrease in child survival for all the countries. It is evidenced in this paper that it is not consistent since the child mortality is bounded. We propose the use of the child survival logit transformation to specify and assess consistently convergence/divergence with regard to human development. Estimates from a weak specification could lead to over-valuate importantly the improvements with regard to health, and more widely to human development, while the efforts have be maintained or even strengthened. An empirical investigation relying on a cross-section analysis over 166 countries and a 30-year period (1970-1999) evidences a very significant absolute divergence. This implies that the Millenium Development Goals expressed in relative

variation, i.e from a log-log specification, should be distinguished according to their initial level.

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APPENDIX 1: origin of under-five mortality data

Data on under-five mortality have been published in the WHO bulletin volume 78, n°10, 2000, by Ahmad, Lopez and Inoue.

Estimates of the probability of dying before the age of five are mostly based on the *World Fertility Survey* (WFS) and *Demographic and Health Survey* (DHS) reports as well as on prior studies, in particular, Hill and Pebley (1989), Hill(1987), United-Nations (1988), (1992), Hill and Yazbeck (1993), Sullivan, Rutstein and Bicego (1994), Bicego and Ahmad (1996), Hill and al.(1999).

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TABLE 1

Three Different Measures of a Same Life Expectancy Improvement

Let A, B, C be 3 countries with a life expectancy of 40, 65 and 75 years respectively, and let be the maximum life expectancy be 85 years. The following table gives three expressions of the improvement obtained for these three countries for a same increase in 6 years.

	A	B	C
Initial level	40	60	75
Distance to the maximum	45	25	10
(1) Absolute increase	6	6	6
(2) Relative increase	15 %	10%	8 %
(3) Relative decrease in the distance to the maximum	13 %	24 %	60 %

To a same absolute increase in life expectancy (1) corresponds an opposite classification depending on whether we consider;

- rate of growth (i.e relative increase) (2)
- performance (i.e "achievement") (3)

Implications of the different measures in analyzing convergence are explained in the text.

TABLE 2- A brief overview of the literature on human development convergence

Auteurs	Ram (1998)	Sab et Smith (2002)	Hobijn et Franses (2002)	Mazumdar (2003)	Neumayer (2004)
Form of convergence	β- convergence : absolute then conditional	β- convergence : absolute then conditional	β- absolute convergence	β- absolute convergence	β- absolute convergence σ- convergence
Aim of the study	Looking for the factors of convergence	Interactions between health and education: analysis of joint convergence between education and health.	Descriptive, only observe convergence, do not try to explain the underlying process	Investigating human well-being convergence/divergence	Assessing the impact of AIDS on convergence/divergence
Reference article		Barro (1991), etc.	Kakwani (1993)	Baumol (1986), Baumol et Wolff (1988)	Hobijn and Franses (2001) Mazumdar (2003)
Explained Variable	$\text{Ln}(\frac{X_t}{X_0})$	$\ln(\frac{X_t}{X_0})$	V(X, M) = ln(M-X) M being the upper bound of the indicator x	$\ln(\frac{X_t}{X_0})$	Rate of growth
Proxy for human status	- Life expectancy	- Life expectancy (life) - School rate (primary, secondary or tertiary, male or female) (SCO)	- Daily caloric ratio - Daily protein ratio - ISR - Life expectancy	- HDI - APQLI - Life expectancy	- Life expectancy - Infant Survival Rate
Dimension	Cross-country 123 countries 1950-90	Cross-country 100 countries; 1975-96	Cross-country 150 countries; 1965-90	Cross-country 98 countries; 1975-99	Cross-country for five ten year periods 186 countries; 1950-60, etc.
Equation to test	$\ln(\frac{X_t}{X_0}) = f(X_O, \ln X_O^2, \ln Y70)$ Y being the log of the GDP pe capita	$\ln(\frac{X_t}{X_0}) = f(\ln X_0, \ln Y_t)$ Y being respectively the school rate and the life expectancy when X is the life expectancy and the school rate	$g_i = \alpha + \beta.X_{i,0} + \varepsilon_i$ with $g_i = \frac{X_{i,T} - X_{i,0}}{T}$	$\ln(\frac{X_t}{X_0}) = f(X_0)$ $\ln(\frac{X_t}{X_0}) = f(X_0, X_0^2)$ $\ln(\frac{X_t}{X_0}) = f(\ln X_0)$ $\ln(\frac{X_t}{X_0}) = f(\ln X_0^2)$ $\ln(\frac{X_t}{X_0}) = f(X_0, X_0^2, \ln X_0)$	$g_i = \alpha + \beta.\ln X_{i,0} + \varepsilon_i$ with $g_i = \frac{X_{i,T} - X_{i,0}}{T}$
Method	OLS	3SLS	OLS	OLS	OLS
Results	Convergence for countries with a life expectancy over 35 years	Convergence for IMR or life expectancy and schooling as well (conditional or not)	Convergence with gross indicators, no more once controlled for the bound	β-convergence for life expectancy on the whole sample	Convergence when controlling for AIDS

TABLE 3 –
Two Measurements of Achievement in Terms of Survival

Antecedent (life expectancy)	Measurement for the survival rate	Approached Measurement when; Max(s) = 1 or min(m)=0	Derivatives characteristics towards s	Denomination
Anand and Ravallion(1993)	$-\ln(M-s_i)$ or $-\ln(m_i - m)$	$-\ln(m_i)$	x'>0 x''>0	Logarithmic achievement
Hobijn and Franses (2001)	id.	id.	id.	id.
Bhalla and Glewwe (1986)	$\ln(\frac{s_i}{M - s_i})$	$\ln \frac{s_i}{m_i}$	x'>0 x''>0 if s>M/2	Logit achievement

NB: both Anand and Ravallion (1993) and Hobijn and Franses (2001) use the distance to the bound (in logs), but only the latter are interested in convergence, the former attempt to assess the impact of different factors but not to assess convergence.

TABLE 4-
Functional form and convergence

Dependent variable	Child survival S	Child survival (in logs) ln(s)		Child mortality (in logs) ln(m)=ln(1000 -s)	Child Survival divided by the distance to its bound (in logs) $\ln(\frac{s}{1000-s})$
Specification	(1)	(2)	(3)	(4)	(5)
	Linear	Log-linear	Log-linear + quadratic term	Log-linear	logit
Good Specification	no	no	no	no	yes
Test of specification: - Godfrey-Wickens t-statistic	-3.34***	2.20**	3.45***	4.04***	-0.14
p-value a)	**0.00**	**0.03**	**0.00**	**0.00**	**0.89**
- Ramsey-Reset F-statistic	8.46***	7.11***	5.21***	5.08***	0.74
p-value a)	**0.00**	**0.00**	**0.00**	**0.00**	**0.53**
Convergence ?	Yes	Yes	N.S	No	No
Coefficient of Convergence b)	0.77*** (0.02)	0.74*** (0.024)	5.05 (0.40)	1.09*** (0.02)	1.16*** (0.03)
Quadratic term			-0.32 (0.30)		

a- the p-value indicates the probability not to reject H_0 that is to conclude on the good-specification.

b- *** statistically significantly different from one at 1%

Robust error standards figures in bracket under the coefficient of convergence

FIGURE 1

Two different views of the evolution of a same human indicator naturally converging to an upper limit: human capital versus performance indices.

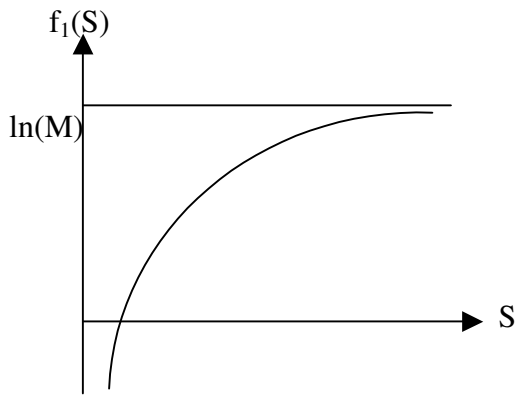
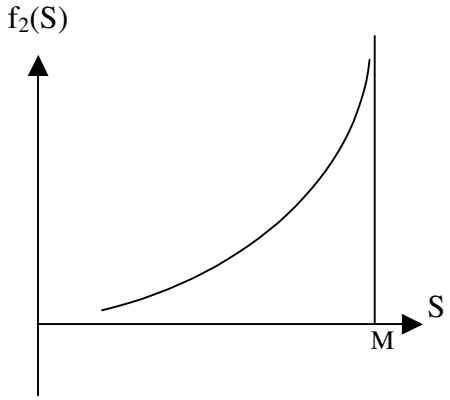
Survival as Human Capital (1)	Survival as Achievement (2)
$f_1(S) = \ln(S)$	$f_2(S) = \ln(\frac{S}{1-S})$
	
(1) The logarithmic transformation of Survival, S, reflects the fact that along its increase, ln(S) naturally converges to an upper limit: its potential marginal impact (on growth) is decreasing: it gives a proxy for a human capital index. $f_1' > 0$ $f_1'' < 0$	*(2) The logistic transformation of Survival reflects the fact that along its increase, the closer to the limit, the harder to get closer: it gives a proxy for a human performance index.* $f_2' > 0$ $f_2'' > 0$

FIGURE 2- Survival Sigma-Convergence: Two Opposite Views

Figure 2.1- Move in the Standard Deviation of the Under-Five Mortality or Survival Rates (1965-1999)

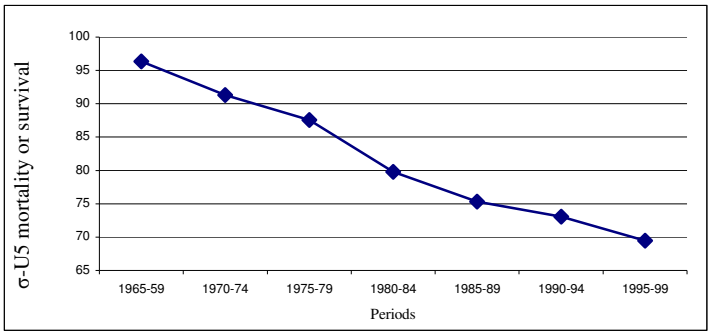


Figure 2.2- Move in the Standard Deviation of the Logarithm of Under-Five Survival Rates (1965-1999)

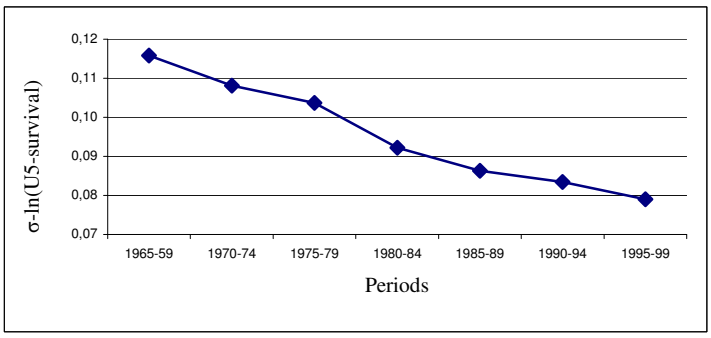


Figure 2.3- Move in the Standard Deviation of the Logarithm of Under-Five Mortality Rates (1965-1999)

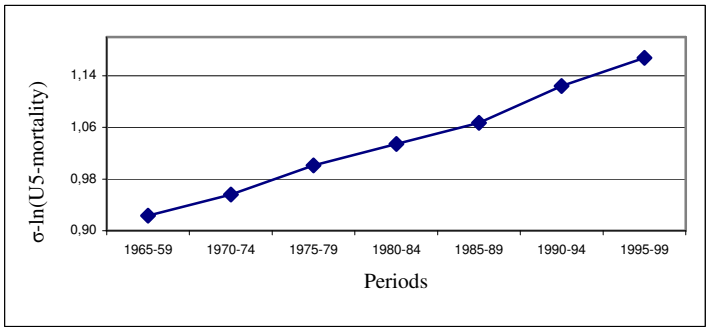
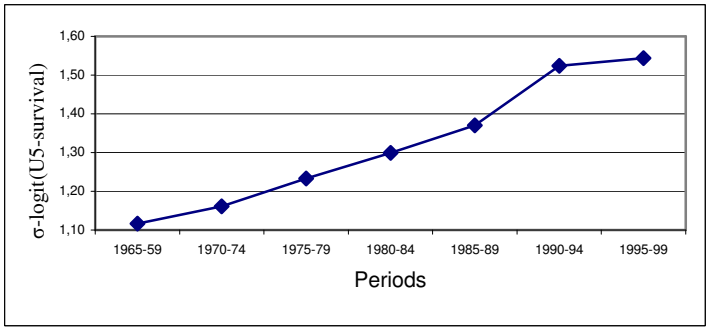


Figure 2.4- Move in the Standard Deviation of the Logit Transformation of Under-Five Survival (1965-1999)



Gross data on 166 countries comes from Ahmad et al.(2000) in the bulletin of the WHO.