





**Abstract**

This paper develops a general equilibrium framework to study the role of preferences structure (additive, multiplicative and convex combination of both) in connecting consumption, health investment, stock of health and capital, and their effects on the wage rate and on productivity. We show that the elasticities of health production, health investment and health cost determine jointly how health influences the wage rate. We examine the steady state and the equilibrium dynamics of the model. In the case of additive preferences, the existence of equilibrium and the stability of the dynamic system require that the ratio of the elasticities of the cost of health and health investment is greater than the elasticity of the production function of health. Health stock can have either positive or negative effects on wage rate. The reverse holds for multiplicative preferences and the effect of health stock on wage rate is always positive. Longevity is a decreasing convex-concave function of the elasticity of inter-temporal substitution of health. We also compare the relative behavior of opportunity costs of health under preferences structure.

**Key words:** Consumption, health investment, preferences structure, wage rates, longevity, opportunity costs

**JEL codes:** C61, C62, I15, E21

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$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma_1} \quad (15a)$$

$$\frac{\dot{m}}{m} = \frac{(-\varphi_M \psi_m^2 + U_C((r - \rho)h_m \psi_m + ((\delta_M + \rho)h_m - w\varphi_M \psi_m)\psi_m))}{U_C m(\psi_m h'_m - h_m)\psi'_m} \quad (15b)$$

$$\frac{\dot{M}}{M} = \frac{bm^\beta}{M} - \delta_M \quad (15c)$$

Proceeding with the final sector, remember that  $f(\hat{k}) = \frac{Y(z)}{L(z)} = B(M(z))\hat{k}(z)^\epsilon$ . Then, the maximization of the profit function under perfect competition allows to equalize the marginal cost of each factor with its marginal benefit. Therefore,

$$r(z) = \epsilon B(M(z))\hat{k}(z)^{\epsilon-1} - \delta \quad (16)$$

$$w(z) = f(\hat{k}(z)) - \hat{k}(z)f'(\hat{k}(z)) = (1 - \epsilon)B(M(z))\hat{k}(z)^\epsilon \quad (17)$$

Combining the demand and the supply sides, we can now characterize the equilibrium of the economy. We can write

$$\dot{\hat{k}}(z) = B(M(z))\hat{k}(z)^\epsilon - \hat{C}(z) - \hat{m}(z) - (\delta + n)\hat{k}(z) \quad (18)$$

where  $\hat{C}(z)$  and  $\hat{m}(z)$  are respectively the consumption and health expenditure per labor, and  $n$  is the population growth rate. Therefore, the dynamics of the economy can be summarized by the following non-trivial four dimensional system:

$$\frac{\dot{\hat{C}}(z)}{\hat{C}(z)} = \frac{r(z) - \rho}{\sigma_1} \quad (19a)$$

$$\frac{\dot{\hat{m}}(z)}{\hat{m}(z)} = \frac{\delta_M + r}{\alpha - \beta} - \frac{b\beta(w + C^{\sigma_1})M^{-\sigma_2}m^{\beta-\alpha}}{\pi\alpha(\alpha - \beta)} \quad (19b)$$

$$\dot{\hat{M}}(z) = bm^\beta - \delta_M \hat{M}(z) \quad (19c)$$

$$\dot{\hat{k}}(z) = B(M(z))\hat{k}(z)^\epsilon - \hat{C}(z) - \hat{m}(z) - \delta\hat{k}(z) \quad (19d)$$

including  $\hat{k}(0)$  and  $\hat{M}(0)$  as given and in addition the transversality conditions. The steady-state values of  $\hat{C}$ ,  $\hat{m}$ ,  $\hat{M}$ , and  $\hat{k}$  are obtained by equalizing  $\dot{\hat{C}}$ ,  $\dot{\hat{m}}$ ,  $\dot{\hat{M}}$ ,  $\dot{\hat{k}}$  to zero. We obtain:

$$\hat{C} = \frac{w}{1 - \epsilon} \left(1 - \frac{\epsilon}{r + \delta}\right) - \left(\frac{\delta_M}{b}\right)^{\frac{1}{\beta}} \widehat{M}^{\frac{1}{\beta}} \quad (20a)$$

$$\hat{m} = \left(\frac{\delta_M}{b}\right)^{\frac{1}{\beta}} \widehat{M}^{\frac{1}{\beta}} \quad (20b)$$

$$\widehat{M}^{\sigma_2 - 1 + \frac{\alpha}{\beta}} = \frac{b\left(\frac{b}{\delta_M}\right)^{-\sigma_2} \beta}{\pi(\delta_M + r)\alpha} \left[ w + \left( \frac{w}{1 - \epsilon} \left(1 - \frac{\epsilon}{r + \delta}\right) - \left(\frac{\delta_M}{b}\right)^{\frac{1}{\beta}} \widehat{M}^{\frac{1}{\beta}} \right)^{\sigma_1} \right] \quad (20c)$$

$$\hat{k} = B(M)^{\frac{1}{1-\epsilon}} \left(\frac{\delta + \rho}{\epsilon}\right)^{\frac{1}{\epsilon-1}} \quad (20d)$$

The following proposition characterizes the solution of the system.

**Proposition 1** *There is a unique solution to Eq.(20c).*

*Proof.* See Appendix A.

The next proposition states the comparative statics of the model as regard the effect of health stock on the wage rate. It also states the conditions of existence for the equilibrium and the stability of the dynamic system.

**Proposition 2** *The effect of health stock on the wage rate is positive provided that  $\alpha \geq \beta(1 - \sigma_2)$  and  $\alpha > \beta$ . Moreover, there exists a minimum wage rate  $w_0$  from which the stock of health impacts positively on the wage rate.*

*Proof.* See Appendix A.

The condition in the proposition means that the ratio of the elasticities of the cost of health and health investment is greater than the elasticity of the production function of health. In addition, for the stock of health to have a positive effect on the wage rate, it is necessary that the wage rate remains higher than a minimum level  $w_0$ . We seek for conditions under which the minimum level  $w_0$  can be determined. Relying on Eq.(A-1; see Appendix A), setting  $\mathcal{W}_N \geq 0$  and  $\mathcal{W}_G > 0$  is equivalent to writing respectively  $w \geq f_1(M)$  and  $w > f_2(M)$  where

$$f_1(M) = \frac{(r + \delta)(1 - \epsilon)}{r + \delta - \epsilon} \left[ \left( \frac{\delta_M}{b} \right)^{\frac{1}{\beta}} M^{\frac{1}{\beta}} + \left( \frac{\left( \frac{b}{\delta_M} \right)^{\sigma_2} \left( \frac{\delta_M}{b} \right)^{-\frac{1}{\beta}} M^{-1 + \alpha - \beta + \beta \sigma_2} \pi (\delta_M + r) \alpha (\beta - \alpha - \beta \sigma_2)}{b \beta \sigma_1} \right)^{-\frac{1}{1 + \sigma_1}} \right]$$

$$f_2(M) = \frac{(r + \delta)(1 - \epsilon)}{r + \delta - \epsilon} \left[ \left( \sigma_1 \frac{\epsilon - r - \delta}{(\epsilon - 1)(r + \delta)} \right)^{\frac{1}{1 - \sigma_1}} + \left( \frac{\delta_M}{b} \right)^{\frac{1}{\beta}} M^{\frac{1}{\beta}} \right]$$

### Insert Figure 1

Figure 1 shows the curves  $f_1(M)$  and  $f_2(M)$ . The dash line curve from the origin becomes solid line from  $M_0$ , while the solid curve from  $w_0$  becomes dash line from  $M_0$ . We have:

$$M_0 = \left[ \frac{\frac{r + \delta - \epsilon}{(1 - \epsilon)(r + \delta)} \left( \frac{b}{\delta_M} \right)^{\frac{1}{\beta} + \sigma_2} \pi (\delta_M + r) \alpha (\alpha - \beta + \beta \sigma_2)}{b \beta} \right]^{\frac{\beta}{1 - \alpha + \beta - \beta \sigma_2}} \quad (21)$$

$$w_0 = \frac{(r + \delta)(1 - \epsilon)}{r + \delta - \epsilon} \left[ \sigma_1 \frac{r + \delta - \epsilon}{(1 - \epsilon)(r + \delta)} \right]^{\frac{1}{1 - \sigma_1}} \quad (22)$$

In fact, the influence of the stock of health on the wage rate is positive in the area bounded by the  $y$ -axis and the solid curve, knowing that the minimum ordinate is  $w_0$ . This domain is  $\sup(f_1, f_2)(M)$ . It is therefore possible that the stock of health has a *negative effect* on the wage rate and it is more related to the specification of the welfare function. Indeed, in the case of additive preferences we find that  $S_{U\varphi}$  and  $V_{SS}$  from Eq.(13) vanish. As a result, the solution  $M^*(z)$  is derived from Eq.(14). Relying on Eq.(7b), the expression of the wage rate is obtained as:

$$w(M^*(z)) = \frac{(\delta_M + r)h_m[m^*(z)]}{\varphi_M[M^*(z)]\psi_m[m^*(z)]} - \frac{S_\varphi[M^*(z)]}{U_C[C^*(z)]S_U[C^*(z)]} \quad (23)$$

where the variables  $m^*(z)$  and  $C^*(z)$  are expressed in function of  $M^*(z)$ .





































































