Implementing Tax Coordination and Harmonization through Voluntary Commitment

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Études et Documents n° 12

June 2016

To cite this document:
Rota-Graziosi G. (2016) “Implementing Tax Coordination and Harmonization through Voluntary Commitment”, Études et Documents, n° 12, CERDI. http://cerdi.org/production/show/id/1815/type_production_id/1
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This work was supported by the LABEX IDGM+ (ANR-10-LABX-14-01) within the program “Investissements d’Avenir” operated by the French National Research Agency (ANR).

Études et Documents are available online at: http://www.cerdi.org/ed

Director of Publication: Vianney Dequiedt
Editor: Catherine Araujo Bonjean
Publisher: Mariannick Cornec
ISSN: 2114 - 7957

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Abstract

Pareto-improving tax coordination, and even tax harmonization, are Nash implementable between sovereign countries without any supranational tax authorities. Following Schelling’s approach, we consider voluntary commitment, which constrains countries’ respective tax rate choices. We develop a commitment game where countries choose their strategy sets in preliminary stages and play consistently during the final one. We determine the set of tax rates, which are implementable by commitment. This allows countries to reach Pareto-improving equilibriums. We also establish that complete tax harmonization may emerge as the subgame perfect Nash equilibrium of the commitment game as long as the asymmetry between countries remains limited. Our analysis contributes to the rationale of tax ranges and, more broadly, of non binding but self-enforcing commitments (not equivalent to cheap talk) in the context of tax competition.

Keywords

Tax competition, Tax coordination, Commitment.

JEL codes

H30, H87, C72.
1 Introduction

How may tax coordination and even harmonization be implemented among tax sovereign countries? This issue is especially puzzling in the economic literature in which sovereignty involves a decentralized equilibrium, while coordination or harmonization often relies on a centralized solution at an international level. A sovereign state displays internal control and external independence from other states. It is the primary decision maker, when it comes to other states. This basic definition of sovereignty justifies the use of game theory to formalize international relations. Consequently, tax competition is usually described through the Nash equilibrium of a simultaneous moves game (see Keen, 2008; or Keen and Konrad, 2012). Tax coordination generally results from a normative approach, through the assumption that some centralization occurs (e.g., a supranational tax agency’s ability to deal with international tax spillovers as in Tanzi, 1999). However, in a world of sovereign states, there is, by definition, no supranational government with the legitimate authority to enforce a regulation on national taxation. Even in the European Union (EU), taxation’s issues remain subject to the unanimity of the member countries, while qualified majority voting applies to many other issues. Any attempt to harmonize or coordinate national tax policy seems to threaten national fiscal sovereignty.

The aim of this paper is to go beyond this deadlock by considering the fact that tax sovereign countries are able to fulfill some commitments, which allow them to implement tax coordination, and even to reach tax harmonization, while remaining the unique tax policy makers and the tax sovereigns. We determine under which conditions tax coordination/harmonization may occur at the Nash equilibrium of a noncooperative game without the delegation of tax power to a supranational authority. Such commitments consist of the voluntary restrictions that countries impose upon their own tax policies. These commitments are nonbinding, but self-enforcing and credible. Countries are able to reach Pareto-superior equilibriums and even under some conditions, to achieve tax harmonization. We develop a metagame, more specifically, a commitment game. During the preliminary stages, the number of which is not limited, both countries announce their respective strategies’ sets and modify them as much as they want. At the final stage, each country chooses its tax rate according to its last commitment. By determining which rates are achievable through this commitment device, we establish that a Pareto-improving tax coordination is Nash implementable, where each country chooses a distinct tax rate. We emphasize that a voluntary commitment to a minimum tax rate is possible.

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1This approach goes beyond the tax competition issue and has been adopted in the debate on the centralized versus decentralized provision of a public good in the presence of interjurisdictional spillovers (see for instance, Oates, 1972; or, more recently, Alesina, Angeloni, and Erszo, 2005).

2The conundrum of sovereignty versus international coordination is not restricted to tax competition. In international trade, for instance, Bagwell and Staiger (2001) consider the Nash equilibrium to be a benchmark world in which sovereign countries have not ratified any international agreements. International agreements are designed to correct inefficiencies resulting from spillover effects across countries. However, these agreements directly compromise national sovereignty.

3In the classical approach of international tax interactions, tax competition is unidimensional: Countries compete in their tax rates only. Tax harmonization is an extreme form of tax coordination, implemented by imposing the same rate among countries.
and Pareto improving, contrasting with previous results of infinitely repeated games of tax competition, where a minimum tax rate reduces the threat of punishment and affects tax cooperation (see Kiss (2012)).

Finally, we consider tax harmonization, where all countries choose the same tax rate. We conclude that harmonization may emerge from a decentralized equilibrium as long as asymmetry among countries remains moderate.

We apply the implementation theory, with commitments under perfect information, to tax competition. We study if and how it is possible to curb tax competition in order to reach a Pareto-improving equilibrium. Our analysis focuses on a specific commitment device consisting of the voluntary restriction of countries’ strategy sets: Each country is able to rule out some actions, i.e. some values of their respective tax rates before tax competition takes place. We then consider if tax coordination and harmonization are Nash implementable through this mechanism, or if the Subgame Perfect Nash Equilibrium(s) (SPNE) is Pareto-superior to the Nash equilibrium of the standard tax competition game (coordination), and if a SPNE may be characterized by a unique value of the equilibrium tax rate (harmonization).

Schelling (1960) highlights the role of commitments in improving players’ outcomes. He extends Stackelberg (1934), who emphasized the advantage of moving first (leading) in a Cournot duopoly. Our analysis follows Schelling’s approach, which appears particularly adequate for studying international interactions, such as tax competition, with respect to von Neumann’s reduction to the normal form of noncooperative game. Indeed, sovereign states have, by definition, the power to make voluntary commitments as emphasized by Myerson (2009). Several authors have recently formalized some of Schelling’s intuitions. Considering a grand reaction function where players choose not only their strategy, but also the set of available actions, Kalai, Kalai, Lehrer, and Samet (2010) establish a commitment folk theorem similar to the repeated game folk theorem. Renou (2009) and Bade, Haeringer, and Renou (2009) develop commitment games, that are actually metagames, where players restrict their strategy sets at preliminary stages, and then play accordingly. The latter establishes a theorem, which reduces the analysis of successful commitments to the study of simple

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4The implementation theory adds mechanisms to a game, such that the equilibrium of the game is socially optimal. We assume here neither the existence of information imperfection, nor that of a social planner in charge of designing the game to be played. Jackson (2001) and Palfrey (2002) provide useful surveys of this theory (with and without information imperfection).

5First, leading corresponds to pure unconditional commitment in Schelling’s terminology, and also serves as one of the commitment technologies studied by Schelling. Second, the existence of a first-mover advantage in the Stackelberg duopoly derives from some properties of the game: Strategic complementarity or substitutability, positive or negative spillovers, etc. For instance, Amir and Stepanova (2006) show that players in a Bertrand duopoly display a second-mover advantage, preferring to follow rather than lead.

6Myerson (2009) writes: “Strategy of Conflict demonstrated both the importance of noncooperative equilibrium analysis and the inadequacy of doing it only in the normal form.” Moreover, in his seminal paper on asymmetric tax competition, Bucovetsky (1991) highlights that the “government may have better ability than colluding firms to make binding commitments.”

7Some authors have formalized commitments in particular games. For instance, Crawford (1982) and Hart and Moore (2004) demonstrate the role of commitment in bargaining, while Hamilton and Slutsky (1990) and van Damme and Hurkens (1996) study the effect of unilateral commitment on a single pure strategy in duopoly.
commitments, where one player engages himself in a unique action at the first stage of the game.\footnote{This theorem has the same implication than the direct revelation mechanism in the information theory in that it simplifies drastically the analysis of relevant commitments to study.} This result makes the leader’s behavior in the Stackelberg game crucial for our analysis, since the committing player behaves as the leader of a Stackelberg: It maximizes a payoff function, which explicitly takes into account the reaction of the follower. However, our analysis departs from a standard Stackelberg game since we do not restrict commitment to moving first or second. We determine which tax rates are Nash implemented through commitment in general, and are Pareto-improving with respect to the standard static tax competition game.

Our analysis contributes to the literature on tax competition/coordination in several ways. First, we adopt a monotone comparative statics approach as formalized by Milgrom and Shannon (1994), and widely applied in industrial organization literature (see Vives, 1999). Assuming positive tax spillovers\footnote{Following Eaton (2004), we will use the term of plain complementarity of tax rates.} and the strategic complementarities of tax rates, we deduce several preliminary results, which hold for many models of tax competition (capital or commodity tax competition). Second, we highlight the point that the perfect sub-game Nash equilibriums of the commitment game are Pareto-superior to the Nash equilibrium of the standard (static) game. The voluntary commitment to rule out some levels of tax rates allows countries to establish some Pareto-improving coordination. Our approach provides some rationale regarding tax ranges as described in Peralta and van Ypersele (2006), or other voluntary restrictions among jurisdictions with tax sovereignty. Third, tax harmonization is Nash implementable through commitments when countries are not too asymmetric.\footnote{Since we distinguish countries by their equilibrium tax rates in the standard [static] tax competition game, our approach allows us to deal with different sources of heterogeneity between countries (size, capital endowment, productivity, preference for public spending, etc.).} Here, harmonization is endogenous and unambiguously Pareto-improving, contrasting with a large part of the related literature, where harmonization is an ad hoc hypothesis involving some redistribution among countries (Kanbur and Keen, 1993; Baldwin and Krugman, 2004; or Zissimos and Wooders, 2008).

This article completes previous work on tax competition. In particular, Persson and Tabellini (1992) develop a political economy model of tax competition, where the decisive (median) voter strategically delegates tax policy to an individual with different preferences. The commitment game we propose here is equivalent in theory to a voluntary strategic delegation game under complete information, where each player/country delegates its taxing power to an agent who has access to a narrower set of actions. Dhillon, Perroni, and Scharf (1999) propose a tax competition framework where the preference for a public good in each jurisdiction is private information. Tax competition may then survive despite tax rate coordination. While their analysis is based on the mechanism design literature under imperfect information, we adopt a different approach by considering a voluntary commitment device under perfect information. Finally, we extend Kempf and Rota-Graziosi (2010) and Kempf and Rota-Graziosi (2014), who apply the endogenous timing game in duopoly,
proposed by Hamilton and Slutsky (1990), respectively to capital and commodity tax competition. Both articles conclude that the Nash equilibrium of the simultaneous game is not commitment robust. However, the studied commitment device remains restricted to single actions (moving early or late) and the issue of tax rate coordination/harmonization is not considered.

The rest of the paper includes the following: section 2 presents the standard tax competition game, its main assumptions, and some preliminary results; section 3 develops the commitment game and our main results; section 4 proposes an analytical illustration of our analysis with quadratic payoff functions, which encompasses several well-known models in the literature on capital tax competition; section 5 concludes.

2 The standard tax competition game

2.1 The set-up of the framework

The canonical framework of tax competition ascribed to Zodrow and Mieszkowski (1986) and Wilson (1986) is a one-period two-country game where a single good is produced from two factors: labor, which is immobile across countries, and capital, which is perfectly mobile. This mobility induces a tax spillover and tax policy interactions between countries.

In order to capture these tax interactions, we consider the following strategic-form game, denoted by $G$.

We define $G = \langle \{1, 2\}, ([0, 1], W^i)_{i \in \{1, 2\}} \rangle$, where the interval $[0, 1]$ is the set of available tax rates for each country and $W^i(t_i, t_j) : [0, 1]^2 \rightarrow \mathbb{R}$ is the payoff function of country $i$. This function may correspond to a welfare function, or collected tax revenue. It is assumed to be continuous in both arguments ($t_i$ and $t_j$) and concave in $t_i$ ($W^i_1(t_i, t_j) \equiv \partial^2 W^i(t_i, t_j) / \partial t_i^2 < 0$).

Following the monotone comparative statics approach, we assume that objective functions and, consequently, the tax competition game, exhibit two properties for any value of the tax rates in $[0, 1]$. The first property is positive tax spillover or, equivalently, the plain complementarity of tax rates as defined by Eaton (2004).

Assumption (1): We assume the plain complementarity of tax rates, or equivalently

$$W^i_2(t_i, t_j) \equiv \frac{\partial W^i(t_i, t_j)}{\partial t_j} > 0. \quad (1)$$

The plain complementarity property sums up several effects, which have been identified in the tax competition literature. For instance, considering the standard capital tax competition game, where the country’s payoff function is its welfare function, involves at least three effects: a tax base effect, a capital income effect
(also called the terms of trade effect), and the productivity effect of immobile factors. The tax base effect corresponds to the following: an increase in country \( j \)'s tax rate decreases the net return of capital in this country and drives out capital from this country into country \( i \). This flow broadens the capital tax base of country \( i \) and increases its tax revenue and payoff function. The capital income effect involves an opposite relationship: due to the perfect mobility of the tax base (capital), an increase in \( t_j \) decreases the worldwide net return of capital, and affects the capital owners in both countries negatively. If the weight of the capital owners in the objective function is sufficiently important,\(^{11}\) we may obtain the plain substitutability property for the capital exporter country \((W_j^i(t_i,t_j) < 0)\). Finally, the productivity effect relies on the shape of the production function: An increase in \( t_j \) raises the level of capital in country \( i \), and then increases the rent from immobile factors, since the production function is usually assumed to be concave. By assuming plain complementarity, we consider that the tax base effect and the productivity effect dominate the capital income effect. Section 4 provides a quadratic illustration of these effects through two models of tax competition one on capital, the other on commodity.\(^{12}\)

The plain complementarity may be considered as a property of first order, since its definition involves a first-order partial derivative of the objective function. Our second assumption, namely the strategic complementarity of tax rates, is a second-order property of the payoff functions and, consequently, of the tax competition game. Here, we follow the definition of Bulow, Geanakoplos, and Klemperer (1985).\(^{13}\)

**Assumption (2):** We assume the strategic complementarity of tax rates, or equivalently

\[
W_{12}^i(t_i,t_j) = \frac{\partial^2 W^i(t_i,t_j)}{\partial t_i \partial t_j} \geq 0. 
\]  

Plain and strategic complementarities are assumed to hold for any values of the tax rates in order that the reaction function of each country be monotone. These hypotheses describe the tax externality, which is often considered in the literature on tax competition. Their combination also creates harmful tax competition, or causes a detrimental race to the bottom, which would require some international tax coordination. Indeed, under plain and strategic complementarities, a decrease in the tax rate of one country induces a negative direct effect on the welfare or tax revenue of the other country, and an indirect effect through the reaction of the other country, which reduces its own tax rate (and then, its revenue) as a best reply.\(^{14}\)

\(^{11}\)The international distribution of capital endowments plays a crucial role here.\(^{12}\)Our main results may also be established with plain substitutability since the commitment game proposed by Bade et al. (2009) does not assumed this property.\(^{13}\)Rota-Graziosi (2016) establish sufficient conditions: the log-concavity and the \(1/2\)-convexity of the marginal production function to establish the strategic complementarity of tax rates in a standard capital tax competition when countries maximize their respective tax revenues. An extension to welfare maximizers is not immediate (see the analysis of Vrijburg and Mooij, 2016, who establish the strategic substitutability of tax rates).\(^{14}\)The race to the top is also possible in this configuration.
Several empirical works (Altshuler and Goodspeed, 2002; Revelli, 2005; Devereux, Lockwood, and Redoano, 2008; Kammas, 2011) confirm the assumption of strategic complementarity by highlighting a positive slope of countries’ reaction functions in tax rates. Plain property has received less attention from the empirical literature. However, a recent analysis (IMF, 2014) estimates the impact of neighboring countries on the Corporate Income Tax (CIT) base for 103 countries over the period 1980-2013. It concludes with a significant and positive spillover base effect, which is equivalent to our assumption of the plain complementarity of tax rates.

If, by contrast, tax rates are plain substitutes and strategic complements, tax competition, which still induces a race to the bottom due to the second property, would be beneficial for both countries, since the decrease in tax rates means an increase in the payoff for both countries. Plain substitutability and strategic complementarity may also be considered as a reduced form of the Brennan and Buchanan (1980) approach, where tax competition is an expedient for taming the Leviathan. However, the Public Choice school usually introduces a critical distinction between the population’s welfare and the payoff of the decision maker, which is beyond the scope of our analysis. The strategic substitutability of tax rates is more debatable in tax competition. This property would prevent the relevance of tax coordination, since a decrease in the tax rate of one country yields an increase in the tax rate of the other.

Given Assumptions (1) and (2), we define the Nash equilibrium, denoted by $t^N = (t^N_1, t^N_2)$, of the tax competition game $G$:

\[
\begin{align*}
    t^N_1 &= \arg \max_{t_1 \in [0,1]} W^1_1 (t_1, t_2) \quad t_2 \text{ given}, \\
    t^N_2 &= \arg \max_{t_2 \in [0,1]} W^2_2 (t_2, t_1) \quad t_1 \text{ given}.
\end{align*}
\]

The First-Order conditions (FOCs) of (3) determine the best reply of each country, denoted by $\tau_i : [0,1] \to [0,1]$ such that

\[
\tau_i (t_j) = \{ t_j \in [0,1] : W^i_1 (\tau_i (t_j), t_j) = 0 \}.
\]

From Assumption (2.1) and the concavity of the $W^i (.)$ with respect to $t_i$, the best reply $\tau_i (t_j)$ is strictly increasing in $t_j$:

\[
\frac{d\tau_i (t_j)}{dt_j} = \left( \frac{W^i_{12} (\tau_i (t_j), t_j)}{W^i_{11} (\tau_i (t_j), t_j)} \right) \geq 0.
\]

An immediate consequence of the strategic complementarity of tax rates is the supermodularity of the tax competition game and the existence of a Nash equilibrium. We have the following Lemma:

**Lemma 1.** Under Assumption (2.1), there is always at least one pure-strategy Nash equilibrium in the tax competition game.

Several authors have studied the existence of a Nash equilibrium in a general standard tax competition model. For instance, Bucovetsky (1991), Wildasin (1991), or Wilson (1991) specified their objective functions in such a way that countries’ best replies are linear and cross only once, ensuring the existence and uniqueness of the Nash equilibrium. Laussel and Le Breton (1998) establish the existence of the Nash equilibrium in a more general framework, but still under some restrictive assumptions: (i) the convexity of the marginal production function, (ii) the linearity of the objective functions in public and private consumption, and (iii) the absence of a capital owner in these functions.

By adopting a monotone comparative statics approach and assuming the strategic complementarity of tax rates, we avoid the difficulties of establishing the existence of a Nash equilibrium. We follow previous analyses in industrial organization, where the notion of supermodularity has proved to be a valuable tool (see the surveys of Amir, 2005; and Vives, 2005). Plain complementarity or substitutability does not affect this result, which continues to hold even when objective functions do not display the usual continuity properties as they do for instance in Kanbur and Keen (1993).

For the sake of simplicity, we will restrict our analysis by assuming the uniqueness of the Nash equilibrium.

Assumption (3): The Nash equilibrium of the game \( G \) is unique.

We now consider the Stackelberg equilibrium of the preceding tax competition game, in which country \( i \) leads and country \( j \) follows. We denote the equilibrium value of this game by \((t^L_i, t^F_j)\). We define the payoff function of the leader \( i \) by \( L^i(t_i) \equiv W^i(t_i, \tau_j(t_i)) \). The result is given by

\[
\begin{align*}
    t^L_i &\equiv \arg\max_{t_i \in [0,1]} L^i(t_i), \\
    t^F_j &\equiv \max\{\min\{\tau_j(t^L_i), 1\}, 0\}.
\end{align*}
\]

We assume an interior solution to the Stackelberg game.

Assumption (4): The payoff function of the leader is concave, or equivalently

\[
\forall t_i \in [0,1], \quad \frac{d^2 L^i(t_i)}{dt_i^2} < 0. \tag{4}
\]

The Stackelberg problem is a special case of the bilevel optimization program, where the leader maximizes its objective function under the constraint of the follower’s maximization program. A significant piece of literature in operational research addresses this issue (see the survey of Colson, Marcotte, and Savard, 2007).
However, to our knowledge, there is no general proof of the existence of an interior solution.15

2.2 Pareto improvement

In this subsection, we define Pareto improvement with respect to the status quo, that being the Nash equilibrium of the standard tax competition game. We proceed in several steps. A preliminary Lemma is established, relying on the concavity of the leader’s objective function.

**Lemma 2.** The function $L^i(t_i) \equiv W^i(t_i, \tau_j(t_i))$ is strictly increasing in $t_i \in [0, t^L_i]$ and strictly decreasing in $t_i \in [t^L_i, 1]$.

*Proof.* This result derives directly from the definition of $t^L_i$ as the global maximum and, from the assumed concavity of $L^i(.)$ (Assumption 2.1). \(\square\)

We then establish a ranking of the equilibrium tax rates, which results from our assumptions of plain and strategic complementarities.

**Lemma 3.** Under Assumptions (1), (2), (3), and (4), there are three possible rankings:

\[
\begin{align*}
&\begin{cases}
  t^N_1 < t^F_1 < t^L_1, \\
  t^N_2 < t^F_2 < t^L_2
\end{cases} \quad \text{and} \quad \\
&\begin{cases}
  t^N_1 < t^L_1, \\
  t^N_2 < t^L_2
\end{cases}
\end{align*}
\]

where $t^F_i \equiv \tau_i(t^L_j)$.

*Proof.* See Appendix A.1. \(\square\)

We note that Nash equilibrium tax rates are always lower than the Stackelberg leader’s tax rate. By determining its tax rate, the leader anticipates the reaction of the follower. Due to the properties of plain and strategic complementarities, any increase in the tax rate of the leader induces the follower to increase its own tax rate. Such a move increases the welfare of the leader because the tax rate of the follower is a plain complement. This ranking has been already established in Kempf and Rota-Graziosi (2010).

Combining Lemmas (2) and (3), we obtain the following

**Lemma 4.** Under Assumptions (1), (2), (3), and (4), we have

\[
\forall t_i \in [t^N_i, t^L_i], \quad W^i(t_i, \tau_j(t_i)) \geq W^i(t^N_i, t^N_j).
\]

15Simaan and Cruz (1973) establish the existence of a Stackelberg equilibrium when the actions’ sets are compact. This result applies to our context since the set of the studied actions is $[0, 1]$. However, it remains inadequate for establishing the existence of an interior solution, since corner solutions may occur ($t^L_i = 0$ or $t^L_i = 1$).
Proof. Immediate from Lemmas (2) and (3).

A country always prefers to lead than to play the simultaneous game. For $t_i = t_i^L$, Lemma (4) corresponds to the first-mover incentive ($W^i(t_i^L, \tau_j(t_i^L)) \geq W^i(t_i^N, t_j^N)$). Lemma (4) may be viewed as an extension of the first-mover incentive.

We then deduce the following Proposition concerning the Pareto improvement with respect to the status quo.

**Proposition 1.** Under Assumptions (1), (2), (3), and (4), a profile of actions $(t_i, t_j)$, such that $t_i \in [t_i^N, t_i^L]$ and $t_j = \tau_j(t_i)$, is Pareto-improving.

**Proof.** For country $i$, we apply Lemma (4).

For country $j$, we have

$$W^j(\tau_j(t_i), t_i) = \max_{t_j \in [0,1]} W^j(t_j, t_i) \geq W^j(t_j^N, t_i) > W^j(t_j^N, t_i^N),$$

where the first inequality is derived from the definition of the reaction function, and the second inequality from the property of plain complementarity ($W^j_2(\ldots) > 0$), as well as from the fact that $t_i > t_i^N$.

Pareto improvement is to be understood with respect to the Nash equilibrium of the standard tax competition game. Proposition 1 contributes to the broader analysis done by Keen and Wildasin (2004), which determines the characterization of Pareto-efficient tax structures applying Motzkin’s theorem. Our result derives directly from the monotone comparative statics and the properties of plain and strategic complementarity, which determine the ranking of tax rates. In the presence of plain substitutes, for instance (and strategic complements), the leader’s tax rate at the Stackelberg equilibrium would be inferior to the Nash equilibrium tax rate.16

3 The commitment game

We now develop a commitment game following Bade et al. (2009) and Renou (2009). We find that countries are able to commit by restricting their future policy choices. Such a commitment is self-enforcing: If country $i$ restricts its tax rate set to $T_i \subset [0,1]$, any action chosen later on must belong to $T_i$. We define a bilateral commitment as a pair $(T_1, T_2)$ where $T_i$ is a non empty, compact, and convex subset of $[0,1]$. Country $i$ commits to choose a tax rate $t_i$ in $T_i = [t_i^L, t_i^R]$, where $t_i^L (t_i^R)$ is the minimum (maximum) of country $i$’s restricted

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action space. The bilateral commitment $T = (T_1, T_2)$ induces the game $G(T) = \left\{ \{1, 2\}, (T_i, W^i)_{i \in \{1, 2\}} \right\}$, which results from game $G$ (defined in the preceding section) by limiting the action set of player $i$ to $T_i$. A unilateral commitment is a particular case of bilateral commitments, where one country does not commit or, equivalently, commits to choose $T_i = [0, 1]$. A unilateral commitment is then equivalent to one of the two Stackelberg games and then the commitment game would correspond to the endogenous timing game studied in Kempf and Rota-Graziosi (2010).

The commitment game, denoted by $G^N(G)$, is a multiperiod game. In each period $n = 1, ..., N$, countries simultaneously make some commitment by restricting their sets of available tax rates $(T^n_i)$. At the final stage, countries play the induced game $G(T^N)$; they determine their tax rates in $T^n_i$ noncooperatively. Bade et al. (2009) establish an equivalence theorem between the multiperiod game and the two-period game. Thus, we will focus our analysis on the following two-stage commitment game, denoted by $G(G)$ ($\equiv G^2(G)$). In the first step, both countries simultaneously choose their tax policy sets $T_i = [t_i, T_i]$. In the second step, they play the induced strategic form game $G(T)$, where $T = T_1 \times T_2$.

A strategy in the game $G(G)$ is a pair $(T_i, \tau_i^{T_i})$, where the set $T_i$ is an interval included in $[0, 1]$, chosen at the first stage of the commitment game, and $\tau_i^{T_i}$ is the induced best reply function of player $i$ to the tax rate of the player $j$, given their respective commitments.

**Lemma 5.** The best reply function of country $i$ is given by

$$
\tau_i^{T_i} : T_j \to T_i \equiv \begin{cases} t_i & \text{if } W_i^j (t_i, t_j) < 0 \\ \tau_i (t_j) & \text{if } W_i^j (\tau_i (t_j), t_j) = 0 \\ \tau_i & \text{if } W_i^j (\tau_i, t_j) > 0 \end{cases}
$$

**Proof.** Country $i$ restricts its tax rate to $T_i = [t_i, T_i]$. If $W_i^j (t'_i, t_j) = 0$ for $t'_i < t_i$, then we have $W_i^j (t_i, t_j) < 0$ for any $t_i > t'_i$, since $W_i^j (t_i, t_j) < 0$. We deduce that $\forall t_i \in T_i$, $W_i^j (t_i, t_j) < 0$. Country $i$ cannot play $t'_i$ due to its commitment and $\forall t_i \in T_i$, $W_i^j (t_i, t_j) \leq W_i^j (t_i, t_j)$. Thus, the best reply to $t_j$, given the commitment on $T_i$, is $\tau_i^{T_i} (t_j) = t_i$. If $W_i^j (t'_i, t_j) = 0$ for $t'_i > T_i$, then $\forall t_i < t'_i$, $W_i^j (t_i, t_j) > 0$. In particular, for any $t_i \in T_i$, we have $W_i^j (t_i, t_j) > 0$, and $W_i^j (t_i, t_j) \leq W_i^j (T_i, t_j)$. Thus, the best reply function of country $i$ to $t_j$ on $T_i$ is $\tau_i^{T_i} (t_j) = T_i$.

Best replies are constrained in the commitment game. These constraints are not imposed exogenously, but they are endogenously chosen at the previous stages of the game. Adopting an alternative approach, Kalai

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17Theorem 2 in Bade et al. (2009) states that for any number of stages $(N)$, a profile of actions $t^*$ is implementable in $G^N(G)$ if and only if it is implementable in $G^2(G)$.
et al. (2010) define the grand reaction function $T_i$, which encompasses the set of available strategies and the restricting best reply as $T_i \equiv \left( T_i, \tau_i^T(t_j) \right)$.

We now determine the set of tax rates, which are Nash implementable through a bilateral commitment, that is, the solution(s) of the preceding commitment game. Following the definition of Bade et al. (2009), we name any bilateral commitment $T_i$, which has the following form $(\{t_i\}, [0, \tau_j(t_i)])$ or $(\{t_i\}, [\tau_j(t_i), 1])$, as simple. These authors establish a theorem, which states that any action profile is implementable through a bilateral commitment if and only if it is implementable by a simple commitment. This theorem significantly simplifies our analysis by allowing us to focus only on simple commitments. We establish the following proposition:

**Proposition 2.** Any tax coordination on rates $(t_i^c, t_j^c)$, with $t_i^c \in [t_1^N, t_1^F]$ and $t_j^c \equiv \tau_j(t_i^c)$, is (i) Nash implementable through bilateral commitment and (ii) Pareto-improving with respect to the status quo.

**Proof.** (i) See Appendix A.2; (ii) Immediate from Proposition (1). \[\square\]

Considering the simple commitment we may provide some insight into this result: Country $i$ commits to the tax rate $t_i^c \in [t_1^N, t_1^F]$, while country $j$ commits to the set $[0, \tau_j(t_i^c)]$. Country $j$ has no incentive to deviate, since it can always play its unconstrained best reply $\tau_j(t_i^c)$ to any committed tax rate $t_i^c$. If country $i$ chooses to deviate from $t_i^c$ to a lower tax rate $t_i^c < t_j^c$, country $j$ continues to play its unconstrained best reply $\tau_j(t_i^c)$, and country $i$, which acts as a leader, would obtain a lower payoff, since, by assumption, the leader's payoff function is concave, thus increasing over $[0, t_1^F]$ and decreasing beyond $(L^1(t_i^c) < L^1(t_i^c))$. If country $i$ chooses to deviate from $t_i^c$ to the higher tax rate $t_i^c$, then country $j$ is constrained in its reaction by its initial commitment. In other words, country $i$ raises its tax rate, while the other does not modify its own policy. This yields a decrease in the payoff of country $i$ ($W_i(t_i^c, \tau_j(t_i^c)) < W_i(t_i^c, \tau_j(t_i^c))$). From Proposition (2), we obtain an immediate Corollary, which defines tax rates as implementable through commitments.

**Corollary 1.** Tax rates $t_i$, such as $t_i < \min \{ t_1^N, t_2^N \}$, or $t_i > \max \{ t_1^F, t_2^F \}$, are not achievable through commitments.

**Proof.** From the proof of Proposition (2), we deduce that the commitment to choose $t_i < t_1^N$ or $t_i > t_1^F$ is not self-enforcing. Indeed, country $i$ is always incited to deviate from its commitment. Similarly, any tax rate $t_i > \max \{ t_1^F, t_2^F \}$ is not self-enforcing. \[\square\]
A minimum tax rate, denoted by \( t_{\text{min}} \), is then implementable through voluntary commitment if \( t_{\text{min}} \in [t_N^1, t_L^2] \cap [t_N^2, t_L^1] \). Such mechanism allows unambiguously a Pareto improving tax coordination. This result highlights a noticeable difference between commitment games and infinitely repeated games of tax competition. Indeed, Kiss (2012) shows how a minimum tax rate would reduce the threat of punishment and jeopardize the sustainability of tax cooperation. Rather, voluntary commitment on a minimum tax rate does not affect countries’ capacity to coordinate.

Beyond tax coordination and minimum tax rates, tax harmonization has been also largely studied in the tax competition literature. It consists of establishing the same tax rate among countries and it appears particularly appealing between sovereign countries. Indeed, by definition, harmonization eliminates tax spillover without any centralization, that is, any delegation of taxing power to a supranational authority. However, harmonization derives usually from an ad hoc assumption in the literature, rarely from government behavior. Using the previous simple commitment mechanism, we determine under which conditions a country can commit to a specific tax rate such that the best reply of the other country corresponds exactly to this same tax rate. Tax harmonization occurs here at the SPNE of our commitment game. We obtain the following Proposition:

**Proposition 3.** A tax harmonization on the tax rate \( t^h \) is Nash implementable by commitment if and only if

\[
\begin{cases}
    t^h \in [t_N^1, t_L^1] \cap [t_N^2, t_L^2] \\
    \exists i \in \{1, 2\}, \text{ such as } \tau_i^{T_i}(t^h) = t^h.
\end{cases}
\]

**Proof.** The first part of the Proposition derives directly from Proposition (2): A harmonized tax rate \( t^h \) is Nash implementable through bilateral commitments if \( (t^h, t^h) \in [t_N^i, t_L^i] \times [t_N^j, \tau_j(t_L^j)] \) or, equivalently, if \( t^h \in [t_N^i, t_L^i] \cap [t_N^j, t_L^j] \). The second part of the Proposition defines the harmonized tax rate. \( \square \)

It is not possible to ascertain the existence of a harmonized tax rate in general. The geometric approach of tax harmonization provides some insight. Harmonization involves identifying the value of the tax rate, where at least one country’s best reply intersects the first diagonal. Appendix A.3 describes two cases: the first one corresponds to the situation, where no country’s best reply intersects the first diagonal, (e.g., \( t_N^1 < t_N^2, t_L^1 < t_L^2 \), and \( t_L^1 < t_L^2 \)), while in the second case, both best replies intersect the first diagonal (e.g., \( t_N^1 < t_N^2, t_L^1 < t_L^2, \) and \( t_L^1 > t_L^2 \)). Assuming that reaction functions are contractions,\(^{19}\) is still insufficient for solving this issue. Indeed, following Banach’s fixed-point theorem, every contraction mapped on a nonempty complete metric space, in this case any interval included in \([0, 1]\), has a unique fixed point. Consequently, the harmonized tax

\(^{19}\)This relies on Assumption [3], relating to the uniqueness of game \( G \)’s Nash equilibrium.
rate $t^h$ exists for each country. However, it may not belong to the interval $I \equiv [t^N_1, t^L_1] \cap [t^N_2, t^L_2]$, and is not always Nash implementable through a bilateral commitment.

From the preceding Proposition, we deduce the following Corollary:

**Corollary 2.** If $[t^N_1, t^L_1] \cap [t^N_2, t^L_2] = \emptyset$, then tax harmonization is not possible through a bilateral commitment.

Here, tax coordination and tax harmonization result from noncooperative behaviors. They are Pareto-improving and self-enforcing through voluntary restrictions. While tax coordination is always feasible as long as our assumptions hold, tax harmonization may be impossible. When countries differ too much in size, initial factor endowments, productivity, or preferences for public goods, any equilibrium tax rates are such that the interval $I$ is empty. An excessive asymmetry prevents tax harmonization.

Proposition (3) and Corollary (2) complete some preceding studies on tax harmonization. For instance, Kanbur and Keen (1993, Proposition 9, Page 886) establish that harmonization with any rate between the Nash equilibrium tax rates harms one of the two countries, typically the smaller one. Wang (1999) obtains a similar conclusion by focusing on harmonization with any rate between the Stackelberg equilibrium rates (Proposition 5, Page 978). Konrad (2009) shows that a nonbinding minimum tax rate, a tax rate below the Stackelberg follower’s, may induce more harmful tax competition. We complete these results by defining which tax rates are Pareto-improving with respect to the Nash equilibrium of the basic game (Proposition 2).

Our results contrast with those of Baldwin and Krugman (2004) and Zissimos and Wooders (2008), which emphasize the distributional property of tax harmonization and conclude with its limited applicability. We show that not only are there some levels of harmonized tax rates for which both countries increase their respective payoffs, but also that these rates are Nash implementable through commitments. Complete harmonization is then possible in a decentralized way, that is at the equilibrium of a noncooperative game. By considering voluntary restriction as the mechanism for achieving tax coordination and even harmonization, we provide a rationale for some existing practices, such as tax rate ranges, minimum and maximum tax rates, etc.

Before presenting a quadratic illustration of our analysis, we raise the question of whether voluntary commitment differs from cheap talk, that is, costless, nonbinding, and nonverifiable messages as defined by Farrell and Rabin, 1996. Indeed, the commitment game we develop is characterized by multiple preplay stages similar to communication games, where players can communicate before choosing their respective actions. This question is related to the scope of our analysis: If self-enforcing commitments are equivalent to cheap talk in the studied game, then any communication between governments is sufficient for establishing Pareto-improving tax coordination. Following the relevant literature, we will say that cheap talk, or costless communication,
effective for solving the tax coordination issue if the tax competition game displays two necessary credibility properties: the self-committing condition as defined by Farrell (1988), and the self-signaling condition, a stronger requirement emphasized by Aumann (1990). We establish the following Corollary:

**Corollary 3.** Under Assumptions (1), (2), (3), and (4), cheap talk is neither self-committing nor self-signaling in the tax competition game.

*Proof.* See Appendix A.4.

We conclude that self-enforcing commitment cannot be reduced to cheap talk, or in other words, cheap talk is ineffective for inducing Pareto-improving tax coordination, while commitment is. Communication between governments on tax policy issues is insufficient for achieving tax coordination.

## 4 Illustrations with linear reaction functions

We apply our analysis to two types of tax competition: commodity tax competition and capital tax competition, where countries’ reaction functions are linear.

### 4.1 Commodity tax competition

The formalization of the commodity tax competition is usually based on a spatial representation of the world à la Hotelling. Let us consider two countries, spatially represented by an interval $[-1, 1]$, that share a border in $b > 0$. Inhabitants of each country purchase goods at home or abroad depending on the difference in tax rates, and on their respective transportation costs. The unitary transport cost is denoted by $d > 0$. In Kanbur and Keen (1993) and Nielsen (2001), each country maximizes its tax revenue. Assuming that country 1 is the smaller, we obtain the following objective functions, which correspond to countries’ tax revenues:

$$W^1(t_1, t_2) = t_1 \left(1 + b + \frac{t_2 - t_1}{d}\right), \quad W^2(t_2, t_1) = t_2 \left(1 - b + \frac{t_1 - t_2}{d}\right).$$

---

20 Authors differ by their measures of the distance function, that is, transport’s cost. For instance, while Kanbur and Keen (1993) consider the absolute value, Nielsen (2001) uses a quadratic function. This latter specification displays the nice property to induce a continuously differentiable objective function. We assert that our analysis remains valid with the specification of Kanbur and Keen (1993), since their game is supermodular.

21 Keen and Konrad (2012) provide an alternative view to this kind of model, where countries compete with regards to tax rate, and firms are able to transfer their profits from one jurisdiction to another, profit-shifting being costly.

The equilibrium tax rates are then:

\[ t^N_1 = \frac{d}{3}(3 + b), \quad t^N_2 = \frac{d}{3}(3 - b), \]
\[ t^L_1 = \frac{d}{2}(3 + b), \quad t^L_2 = \frac{d}{2}(3 - b). \]

Tax harmonization is possible and implementable through commitments in the Nielsen model, under the condition that the two countries do not differ dramatically in size, more specifically \( b \leq 1/3 \). Thus, under this condition, the unique harmonized tax rate \( t^h = d(1 + b) \) is implementable through the following simple bilateral commitment: country 2 (the smaller country) commits to \( t^h \), country 1 commits choosing its tax rate in \( T_1 = [0, \tau_1(t^h)] \). Since \( \tau_1(T_1(t^h)) = t^h \), tax harmonization occurs and is Pareto-improving:

\[
\begin{align*}
W^1(t^h, t^h) &= (1 + b)^2 d > (1 + b/3)^2 d = W^1(t^N_2, t^N_1), \\
W^2(t^h, t^h) &= (1 - b^2) d > (1 - b/3)^2 d = W^2(t^L_2, t^L_1) \text{ for } b \in [0, 1/3].
\end{align*}
\]

When countries differ moderately in size \((1/3 < b < 3/5)\), tax harmonization is not reachable through voluntary commitments, while Pareto-improving tax coordination remains possible on the following tax rates: \((t^1_1, t^2_2) \in [d(3 + b)/3, d(3 + b)/2] \times [d(3 - b)/3, d(3 - b)/2] \). Finally, when asymmetry is sufficiently important \((b > 3/5)\), commitment is useless for achieving any tax coordination. In other terms, the Nash equilibrium of the standard commodity tax competition game, as described in Nielsen (2001), is commitment robust.

### 4.2 Capital tax competition

We consider a two-country version of the workhorse model of the international capital tax competition proposed by Keen and Konrad (2013). Two countries compete by tax rate to attract capital, which is perfectly mobile between them and is fixed in total supply. The representative resident in country \( i \) is endowed with a share \( \theta_i \) of worldwide capital, denoted by \( \overline{k} \). We have \( \forall i \in \{1, 2\}, 0 \leq \theta_i \leq 1, \) and \( \theta_1 + \theta_2 \leq 1 \). A single homogenous good is produced in each country using inputs of labor and capital. The representative resident in each country supplies one unit of labor. The production function, which is assumed to be concave in its two inputs, and homogenous of degree one, is represented in terms of the capital-labor ratio, denoted by \( k_i \).

We consider the following quadratic form: \( f_i(k) = (a_i - bk) k \), with \( a_1 > a_2 > 0 \) and \( b_i > 0 \).

The perfect mobility of capital involves the following capital market clearing conditions:

\[
\begin{align*}
&\left\{ \begin{array}{l}
f'_i(k_1) - t_1 = f'_i(k_2) - t_2 = r \\
k_1 + k_2 = \overline{k}
\end{array} \right.,
\end{align*}
\]

\(^{23}\text{Plain and strategic complementarities are always respected.}\)
where \( r \) is the net return of capital. The total income of the representative inhabitant in country \( i \) is then

\[
x_i = f_i(k_i) - f_i'(k_i) k_i + r \theta_i \overline{k}.
\]

Capital is taxed at the source, and all government revenue is spent on the public good: \( g_i = t_i k_i \). We simplify our analysis by assuming a constant marginal rate of substitution between private and public consumption, which is equal to \( 1 + \lambda_i \). The objective function of each country is given by \( U_i(x_i, g_i) \equiv x_i + (1 + \lambda_i) g_i \). Substituting \( x_i \) and \( g_i \) by their previous respective expression we obtain

\[
W^i(t_i, t_j) = f_i(k_i) - f_i'(k_i) k_i + r \theta_i \overline{k} + (1 + \lambda_i) t_i k_i.
\]

Given the market clearing conditions (5) reaction functions are defined by

\[
t_i(t_j) = \arg \max_{t_i \in [0,1]} W^i(t_i, t_j) = \frac{3}{3 + 4 \lambda} \left[ (1 + 2 \lambda) t_j + (1 + 2 \lambda) (a_i - a_j + 2b \overline{k}) + b \theta_i \overline{k} \right].
\]

For the sake of simplicity, we consider the following values of the parameters: \( \lambda = 1 \) and \( \overline{k} = 1 \). Relevant equilibrium tax rates are

\[
\begin{align*}
t_i^N &= \frac{3(a_i - a_j) + b(15 - 7 \theta_i - 3 \theta_j)}{10}, \\
\bar{t}_i^L &= \frac{12(a_i - a_j) + b(60 - 35 \theta_i - 12 \theta_j)}{26}.
\end{align*}
\]

Pareto-improving tax coordination is possible as long as \( t_i^N < t_j^L \) or, equivalently,

\[
\frac{a_i - a_j}{b} > \frac{105 + 31 \theta_i - 136 \theta_j}{99}.
\]

Moreover, if \( a_1 > a_2 \), a harmonized tax rate equal to \( t^h = \frac{3}{4} (a_1 - a_2 + 2b) + b \theta_1 \) may be implemented through commitment.\(^{24} \) We turn to numerical simulations since the number of parameters involves tedious expressions.\(^{25} \) Figure 1 displays countries’ reaction functions for the following set of parameters: \( a_1 = 129/128 > a_2 = 1, b = 1/4, \overline{k} = 1, \) and \( \theta_1 = 1/32 < \theta_2 = 31/64 \). The blue area represents the commitment game’s set of equilibrium tax rates: \((t_1^N, t_2^L) \in [t_1^N, t_1^L] \times [t_2^N, t_2^L] \). Tax harmonization occurs when the first bisector \((t_2 = t_1)\) intersects country 1’s reaction function curb in \( t^H \).\(^{26} \) In contrast, Figure 2 displays countries’ reaction functions with \( a_1 = 159/107 > a_2 = 1, b = 1/4, \overline{k} = 1, \) and \( \theta_1 = 3/655 < \theta_2 = 7/1189 \). Tax harmonization through commitment is impossible, since \((t^H, t^H) \notin [t_1^N, t_1^L] \times [t_2^N, t_2^L] \).

\(^{24} \)If \( a_1 < a_2 \), the harmonized tax rate is \( t^h = \frac{3}{4} (a_2 - a_1 + 2b) + b \theta_2 \).

\(^{25} \)A Mathematica file presenting our computations is available upon request.

\(^{26} \)We note that \((t^H, t^H) \in [t_1^N, t_1^L] \times [t_2^N, t_2^L] \).
5 Conclusion

We consider countries’ commitment capacities in the context of international tax competition. The studied commitment technology involves voluntary restrictions on tax rates. Under the assumptions of plain and
strategic complementarities, the Nash equilibrium tax rates are lower than those fixed by the leader in the two Stackelberg games. These rates determine the set of tax rates, on which any coordination through bilateral commitments is mutually beneficial. We also show that a complete tax harmonization is Nash implementable and Pareto-improving as long as the asymmetry between countries remains limited. Analytical illustrations allow us applying our results to the quadratic specification of commodity tax competition and capital tax competition.

Voluntary commitment is an integral part of transgovernmentalism and other new governance methods on the international scene. Studying the European economic and political integration processes, Majone (2005) highlights the obsolescence of the Community method and its standard federalist approach, which essentially consists of delegating competences to a supranational institution. He concludes that there is a need for alternative methods by which member states can credibly commit themselves to a collective action. One of them is the Open Method of Co-ordination (OMC) defined by the Lisbon Council of March 2000. It consists of soft law mechanisms, such as guidelines, peer reviews, benchmarking, and the sharing of best practices. Establishing nonbinding but self-enforcing rules appears to be a viable alternative. These rules avoid several of the pitfalls of binding agreements, such as the bargaining of content and the monitoring of effective enforcement. Illustrations of potential nonbinding political commitments in tax competition are the Code of Conduct for business taxation signed by the EU Member States in December 1997, the OECD initiative undertaken in 1998 to limit "harmful tax competition" or the more recent OECD/G20 Base Erosion and Profit Shifting project for curbing avoidance by multinational enterprises. These examples try to support some regional or global tax regulation. However, the issue remains to determine (empirically) if such initiatives or projects are only cheap talk or effective voluntary commitments.

Several restrictions limit the scope of our analysis. First, critics of the standard tax competition apply here, too. In particular, the studied tax policy remains unidimensional, apprehended through tax rates only, while the multidimensionality of national tax system (e.g., tax base definition and tax policy enforcement) matters a great deal, as stressed by Robinson and Slemrod (2012). Public spending is also a key element of fiscal competition.

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27 The agreement on a minimum rate of 15 percent for the value-added tax (VAT) between EU member states may be considered as an example of such an approach in tax policy. However, the coordination of the European VAT system remains far from complete.

28 Ania and Wagener (2013) present the OMC as an imitative learning dynamic, where some countries mimic best practices already in place in others. They use the notion of evolutionary stable strategies and highlight the fact that some coordination occurs in a subset of Nash equilibriums. However, this subset is not linked to efficient outcomes. In other words, the OMC, as a learning dynamic, does not systematically lead to a Pareto-superior equilibrium.

29 For Radaelli (2003) the Code of Conduct may have induced some convergence in the discussions between the finance ministers regarding harmful tax competition. However, this convergence in finance ministers' speeches does not mean a convergence of actual tax policy.

30 Public spending is also a key element of fiscal competition.
tax system. Second, our assumptions of plain and strategic complementarities, and the uniqueness of the Nash equilibrium, are restrictive. They may be relaxed, since the theorem of Bade et al. (2009) holds for more general forms of the reaction functions. Finally, one of the main constraints is the number of players, which is limited to two. Developing an $n$-player commitment game remains an issue for future research. The application of the notion of the grand reaction function, as defined by Kalai et al. (2010), would be useful.

The purpose of our analysis is not to formalize actual negotiations among countries for achieving some tax coordination. Following the implementation theory, our approach remains mainly normative. The simple commitment mechanism we consider is a convenient tool for identifying which and how countries may coordinate and even harmonize. We stress that By establishing the point that coordination and harmonization may be achievable within a decentralized equilibrium, we state that supranational authority is not indispensable to international coordination. Successive nonbinding but self-enforcing commitments would be useful in attaining a Pareto-superior equilibrium. These commitments are not equivalent to cheap talk; they are more than that (see Corollary 3). Further research may be devoted to developing a successful path of commitments through mutual adjustments, which involves Pareto-improving coordination. Finally, our approach may be useful for formalizing interactions between sovereign countries in contexts other than that of tax competition, for example in international trade, environmental economics, or macroeconomic models, where coordination failures prevail (see Cooper and John (1988)).
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Appendix

A.1 Lemma (3)

From the definition of the Stackelberg and Nash equilibriums, we always have

\[ L^i (t_i^L) = W^i (t_i^L, \tau_j (t_i^L)) = \max_{t_i \in [0,1]} W^i (t_i, \tau_j (t_i)) \geq W^i (t_i^N, \tau_j (t_i^N)) = W^i (t_i^N, t_j^N). \]  \tag{7}

We establish that \( t_j^N < \tau_j (t_i^L) \). Let us consider by contradiction, that \( t_j^N \geq \tau_j (t_i^L) \equiv t_j^F \). In that case, the plain complementary assumption would yield

\[ W^i (t_i^N, t_j^N) = \max_{t_i \in [0,1]} W^i (t_i, t_j^N) \geq W^i (t_i^L, t_j^N) > W^i (t_i^L, t_j^F), \]

which contradicts (7). Thus, we always have,

\[ \tau_j (t_i^L) = t_i^F > t_j^N, \quad i = A, B. \]  \tag{8}

The FOC of the leader’s maximization program in the Stackelberg game is

\[ \frac{dL^i(t_i)}{dt_i} = 0 \iff \left. W_1^i (t_i^L, \tau_j (t_i^L)) + W_2^i (t_i^L, \tau_j (t_i^L)) \frac{d\tau_j (t_i)}{dt_i} \right|_{t_i^L} = 0. \]

From the plain and strategic complementarity assumptions, we have

\[ W_2^i (t_i^L, \tau_j (t_i^L)) \left. \frac{d\tau_j (t_i)}{dt_i} \right|_{t_i^L} \geq 0, \]

which induces

\[ W_1^i (t_i^L, \tau_j (t_i^L)) \leq 0 = W_1^i (t_i^N, t_j^N). \]  \tag{9}

From the concavity of \( W^i (t_i, t_j) \) with respect to \( t_i \), inequalities (8) and (9) involve

\[ t_i^L \geq t_i^N. \]  \tag{10}

Applying backward induction to solve the Stackelberg game, the FOC of the follower is

\[ \forall t_j \in [0,1], \quad W_1^i (t_i^F, t_j) = 0. \]  \tag{11}
From (9) and (11), we deduce that
\[
W_1^i(t_{i}^{L},t_{j}^{F}) \leq W_1^i(t_{i}^{F},t_{j}^{L}) = 0.
\] (12)

We consider two cases:

If \(t_{i}^{L} < t_{i}^{F}\), the concavity of \(W^i(.)\) with respect to \(t_i\) involves
\[
W_1^i(t_{i}^{L},t_{j}^{F}) > W_1^i(t_{i}^{F},t_{j}^{F}) .
\]

From (12), we obtain
\[
W_1^i(t_{i}^{F},t_{j}^{F}) < W_1^i(t_{i}^{F},t_{j}^{L}) ,
\]
and the strategic complementarity of \(W^i(.)\) yields
\[
t_{j}^{F} < t_{i}^{L} .
\]

If \(t_{i}^{L} > t_{i}^{F}\), inequality (12) and the concavity of \(W^i(.)\) with respect to \(t_i\) involve an ambiguous result:
\[
t_{j}^{F} \geq t_{j}^{L} .
\]

Thus, three possible rankings are possible:

\[
\begin{cases}
    t_{i}^{N} < t_{i}^{F} \leq t_{i}^{L} , \\
    t_{j}^{N} < t_{j}^{F} \leq t_{j}^{L} , \\
    t_{j}^{N} < t_{j}^{F} < t_{j}^{L} ,
\end{cases}
\]

\[
\begin{cases}
    t_{i}^{N} < t_{i}^{F} < t_{i}^{L} , \\
    t_{i}^{N} < t_{i}^{F} \leq t_{i}^{L} , \\
    t_{i}^{N} < t_{i}^{F} \geq t_{i}^{L} ,
\end{cases}
\]

\[
\begin{cases}
    t_{j}^{N} < t_{j}^{F} < t_{j}^{L} , \\
    t_{j}^{N} < t_{j}^{F} \leq t_{j}^{L} , \\
    t_{j}^{N} < t_{j}^{F} \geq t_{j}^{L} .
\end{cases}
\]

**A.2 Proof of Proposition (2)**

**A.2.1 Preliminary results**

Several preliminary results may be deduced from the three assumptions we have made. They will be useful in establishing our main Proposition.

**Lemma 6.** We have the following expressions:

For \(t_i \leq \tau_i(t_j)\), \(W_1^i(t_i,t_j) \geq 0\),

For \(t_i > \tau_i(t_j)\), \(W_1^i(t_i,t_j) < 0\).
This Lemma derives from the definition of the best reply function and the concavity of $W^i(t_i, t_j)$ with respect to $t_i$.

**Lemma 7.** We have the following expressions:

$$
\begin{align*}
\forall t \in [0, t_i^N], & \quad \tau_i(\tau_j(t)) > t, \\
\forall t \in ]t_i^N, 1[, & \quad \tau_i(\tau_j(t)) < t, \\
\text{if } t = t_i^N, & \quad \tau_i(\tau_j(t)) = t.
\end{align*}
$$

**Proof.** Following Bade et al. (2009) we establish this result by contradiction. According to the definition of the Nash equilibrium, we have $\tau_i(\tau_j(t_i^N)) = \tau_i(t_i^N) = t_i^N$. The function $\tau_i(\tau_j(t)) - t$ is zero at the Nash equilibrium only, and is either positive or negative for $t \neq t_i^N$. Assume that $\tau_i(\tau_j(t)) - t < 0$ for any $t < t_i^N$.

We then have for $t = 0$, $\tau_i(\tau_j(0)) - 0 = \tau_i(\tau_j(0)) < 0$, which contradicts $\tau_i(t) \geq 0$ for any $t$. $\square$

**Lemma 8.** $W^i(t, \tau_j(t_i))$ is increasing in $t \in [0, \tau_i(\tau_j(t_i))]$ and decreasing in $t \in [\tau_i(\tau_j(t_i)), 1]$.

**Proof.** Let us consider $W^i(t, \tau_j(t_i))$. The definition of the reaction function $\tau_i(\cdot)$ involves

$$W^i_1(\tau_i(\tau_j(t_i)), \tau_j(t_i)) = 0.$$ 

From the concavity of $W^i_{11}(\cdot, \cdot)$ with respect to $t_i$, we deduce that

$$W^i_1(t, \tau_j(t_i)) > 0 \iff t < \tau_i(\tau_j(t_i)),$$

$$W^i_1(t, \tau_j(t_i)) < 0 \iff t > \tau_i(\tau_j(t_i)).$$

$\square$

### A.2.2 Proof of the proposition

Consider that country $i$ commits to a level of tax rate $t_i \in [t_i^N, t_i^L]$ and country $j$ commits to an interval of tax rate $[0, \tau_j(t_i)]$.

1. First, we establish that country $j$ has no incentive to deviate from its commitment. If country $i$ commits to $t_i$ and plays $t_i$, country $j$ is able to play its unconstrained best reply, maximizing its welfare level:

$$\tau_j(t_i) = \arg \max_{t_i \in [0,1]} W^j(t_j, t_i).$$
2. Secondly, we establish that country $i$ has no incentive to deviate from its commitment $\{t_i\}$ if and only if $t_i \in [t_i^N, t_i^L]$. Following Bade et al. (2009), we focus on simple commitments, in particular, country $i$'s commitment to $\{t_i\}$ and country $j$'s commitment to $[0, \tau_j(t_i)]$.

(a) When country $i$ commits to a level of tax rate $t_i \in [t_i^N, t_i^L]$, it should have no incentive to deviate.

i. If $t_i' < t_i$, then $\tau_j^{[0, \tau_j(t_i)]}(t_i') = \tau_j(t_i')$. From $t_i \in [t_i^N, t_i^L] \subset [0, t_i^L]$, it yields

$$L^i(t_i') = W^i(t_i', \tau_j(t_i')) < W^i(t_i, \tau_j(t_i)) = L^i(t_i),$$

since $L^i(.)$ is increasing over $[0, t_i^L]$ (see Lemma 2), and $t_i' < t_i \leq t_i^L$.

ii. If $t_i' > t_i$, then $\tau_j^{[0, \tau_j(t_i)]}(t_i') = \tau_j(t_i)$. From Lemma (7), we have $t_i > \tau_i(\tau_j(t_i))$ and we know from Lemma (7) that $W^i(t, \tau_j(t_i))$ is decreasing in $t$ for all $t > \tau_i(\tau_j(t_i))$. In considering $t_i' > t_i > \tau_i(\tau_j(t_i))$, we deduce that

$$W^i(t_i', \tau_j(t_i)) < W^i(t_i, \tau_j(t_i)).$$

(b) When country $i$ commits to a level of tax rate $t_i < t_i^N$, we show that it has an incentive to deviate from this level in the commitment game $\Gamma(G)$. Since $t_i < t_i^N$, we have from Lemma (7): $t_i < \tau_i(\tau_j(t_i))$. Let consider $t_i'$ such that $t_i < t_i' < \tau_i(\tau_j(t_i))$, we have $\tau_j^{[0, \tau_j(t_i)]}(t_i') = \tau_j(t_i)$ since $t_i < t_i'$. From Lemma (8), we deduce that

$$W^i(t_i', \tau_j(t_i)) > W^i(t_i, \tau_j(t_i)).$$

Country $i$ has an incentive to deviate from $t_i$ by fixing $t_i' > t_i$.

(c) When country $i$ commits to a level of tax rate $t_i > t_i^L$, we show that it has incentive to deviate from this level in the commitment game $\Gamma(G)$. Consider $t_i' < t_i$, then $\tau_j^{[0, \tau_j(t_i)]}(t_i') = \tau_j(t_i)$. Since the function $L^i(.)$ is decreasing over $[t_i^L, 1]$, we deduce that:

$$L^i(t_i') > L^i(t_i).$$

Country $i$ has an incentive to deviate from $t_i$ by fixing $t_i' < t_i$. 


A.3 On the existence of an harmonized tax rate

Let us consider that $t^N_1 < t^N_2$, $t^L_1 < t^L_2$, and $t^L_1 > t^L_2$. We have $I = [t^N_1, t^L_1] \cap [t^N_2, t^L_2] = [t^N_2, t^L_1]$. We establish that no best reply intersects the first diagonal over the interval $I$. By assumption, we have

$$\tau^I_1 (t^N_2) = t^N_1 < t^N_2.$$ 

From Lemmas (7) and (3), we deduce that

$$t^L_1 > t^N_1 \Rightarrow t^L_1 > \tau^I_1 (\tau^I_2 (t^L_1)) = \tau^I_1 (t^L_2).$$

Since $\tau^I_1 (.)$ is increasing, and $t^L_2 > t^L_1$, by assumption, we obtain

$$t^L_1 > \tau^I_1 (t^L_2) > \tau^I_1 (t^L_1).$$

We conclude that the best reply of country 1 is always below the first diagonal on interval $I$:

$$\forall t \in I, \quad \tau^I_1 (t) < t. \quad (13)$$

Let us consider country 2. By assumption, we have

$$\tau^I_2 (t^N_1) = t^N_2 > t^N_1.$$ 

Similarly, by assumption, we have

$$\tau^I_2 (t^L_1) = t^F_2 > t^L_1.$$ 

Thus, we deduce that the best reply of country 2 is also always below the first diagonal on interval $I$:

$$\forall t \in I, \quad \tau^I_2 (t) < t.$$ 

Consider $t^N_1 < t^N_2$, $t^L_1 < t^L_2$, and $t^L_1 > t^L_2$. We have $I = [t^N_2, t^L_1]$. First, by assumption, we have

$$\begin{cases} 
\tau^I_2 (t^N_1) = t^N_2 > t^N_1 \\
\tau^I_2 (t^L_1) = t^F_2 < t^L_1 
\end{cases}.$$
In other words, the best reply of country 2 intersects the diagonal on interval $I$. For country 1, we have

$$\tau_1^I (t_2^N) = t_1^N < t_2^N.$$ 

From Lemmas (7) and (3), we deduce that

$$t_1^L > t_1^N \Rightarrow t_1^L > \tau_1^I (\tau_2^I (t_1^L)) = \tau_1^I (t_2^L).$$

Since $\tau_1^I (.)$ is increasing, and, by assumption $t_2^N < t_1^L$ we obtain

$$\tau_1^I (t_2^N) < \tau_1^I (t_1^L).$$

Thus, we have

$$\tau_1^I (t_1^L) \geq t_1^L.$$ 

The best reply of country 1 may intersect the diagonal.

A.4 Corollary (3)

Following Baliga and Morris (2002), who study games with plain and strategic complementarities in industrial organization, we consider the following definitions.

**Definition 1.** Action $t_i$ is self-committing if

$$W^i (t_i, \tau_j (t_i)) > W^i (t'_i, \tau_j (t'_i)) \text{ for } \forall t'_i \in [0, 1].$$

By definition, only the leader’s tax rate at the Stackelberg equilibrium $t_1^L$ is self-committing on $[0, 1]$.

**Definition 2.** Action $t_i$ is self-signaling if

$$W^i (t_i, \tau_j (t_i)) > W^i (t'_i, t_j) \text{ for } \forall (t'_i, t_j) \in [0, 1]^2.$$ 

Given the plain complementarity, we have

$$W^i (t_i, \tau_j (t_i)) < W^i (t_i, t'_j) \text{ for } t'_i > \tau_j (t_i).$$
Under the assumption of plain and strategic complementarities, each country is always incited to induce the other country to raise its tax rate. The self-signaling condition does not hold here.

\[31\] These results have already been established by Baliga and Morris (2002) in industrial organization.